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ONLINE APPENDIX TO
Foreign Shocks as Granular Fluctuations*

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Appendix A Data and Reduced-Form Results

A.1 Harmonizing French Firm-Level Data with Global Sectoral Data

The firm's sector in the French data is reported in the *Nomenclature d'Activités Françaises* classification, which we convert into the 35 sectors of the WIOD nomenclature. Note that the balance-sheet data do not cover Financial Activities and Private Households with Employed Persons (sectors J and P in WIOD), and thus those sectors are dropped from the analysis. We also dropped the "Public Administration" sector (sector L) which represents 23 firms and less than 0.02% of overall value added in our data. Data on individual bilateral imports, together with information on each firm's cost structure, are used to recover the firm-specific input shares.

Figure A1(a) plots the cumulative distribution function of firm-level share of exports in total sales. Similarly, Figure A1(b) plots the distribution of the intensity of imported input use, summarized by the share of foreign inputs in firms' total input expenditure ($\sum_{n\neq m}\sum_{i\in T}\pi^{M}_{f,mn,ij}$). In both plots, the solid (red) line depicts the unweighted distribution and the (blue) circles the distribution weighted by the firms' share in overall value added.

We stress two features of these figures, both of which are known in the trade literature and are confirmed in our data. First, there is a great deal of heterogeneity across firms in both export intensity and imported input use. Second, participation in foreign markets is heavily tilted towards larger firms. This is illustrated in Figure A1 by the comparison between the weighted and unweighted distributions. In both cases, the cdfs of the weighted distributions are substantially below the unweighted ones, meaning that on average larger firms have higher export and import intensities. In unreported results, we checked that the heterogeneity is not driven by cross-sector differences in overall exposure. While non-traded good sectors tend to be relatively less dependent on foreign inputs, most of the heterogeneity is actually driven by the within-sector variation.

Firm-specific labor shares $\pi^{l}_{f,n,j}$ are defined as the ratio of value added over sales, both available in the balance-sheet data. In order to ensure comparability with the rest of the sample, in which labor shares are calibrated using WIOD for each country and sector, the distribution of firm-level labor shares is rescaled sector-by-sector in a way that preserves the heterogeneity but ensures that the weighted average across firms matches the corresponding information in the WIOD. Namely:

$$\pi^{l}_{f,n,j} = \tilde{\pi}^{l}_{f,n,j}\frac{\pi^{l}_{n,j}}{\tilde{\pi}^{l}_{n,j}}.$$

In this equation, $\pi^{l}_{f,n,j}$ and $\tilde{\pi}^{l}_{f,n,j}$ are the rescaled and original firm-level coefficients, respectively, and $\pi^{l}_{n,j}$ is the sectoral counterpart recovered from the WIOD data. Finally, $\tilde{\pi}^{l}_{n,j}$ is a weighted average of the original firm-level coefficients, where each firm is weighted according to its share $\omega^{S}_{f,n,j}$ in sectoral sales: $\tilde{\pi}^{l}_{n,j} = \sum_{f\in(n,j)}\omega^{S}_{f,n,j}\tilde{\pi}^{l}_{f,n,j}$.[1]

[1]The rescaling procedure implies that some rescaled firm-level coefficients end up lying outside of the range of

Table A1. Summary Statistics of Firms, by Sector

WIOD sector	# firms	Share VA	Traded/non-traded
Agriculture	1,146	0.0008	T
Mining	1,047	0.0051	T
Food products	12,257	0.0434	T
Textile products	4,143	0.0102	T
Leather & Footwear	598	0.0021	T
Wood products	2,933	0.0050	T
Paper products	6,339	0.0142	T
Refined petroleum	47	0.0080	T
Chemical products	2,231	0.0437	T
Rubber and Plastics	3,199	0.0190	T
Other Non-Metallic Mineral	2,812	0.0150	T
Basic and Fabricated Metals	13,061	0.0400	T
Machinery. Nec	4,525	0.0220	T
Electrical Equipment	3,444	0.0333	T
Transport Equipment	1,546	0.0287	T
Manufacturing. Nec	12,147	0.0226	T
Electricity. Gas and Water Supply	2,250	0.0436	NT
Construction	61,761	0.0730	NT
Retailing Motor Vehicles	26,150	0.0348	NT
Wholesale trade	54,899	0.1045	NT
Retail trade	83,606	0.0903	NT
Hotels and Restaurants	32,412	0.0295	NT
Inland Transport	10,101	0.0422	NT
Water Transport	195	0.0021	NT
Air Transport	70	0.0091	NT
Other Transport Activities	2,433	0.0227	NT
Post and Telecommunications	10,192	0.1049	NT
Real Estate Activities	8,596	0.0189	NT
Business Activities	28,675	0.0772	NT
Education	2,018	0.0025	NT
Health and Social Work	6,310	0.0187	NT
Other Personal Services	16,514	0.0128	NT
Total	417,657	1.0000	

Notes: This table reports summary statistics on the numbers of firms and shares of aggregate value added by WIOD sector. The data are from INSEE-Ficus/Fare and correspond to year 2005.

possible values ($[0, 1]$). The corresponding coefficients are winsorized at the maximum and minimum values. This affects less than 2% of the firms in the total sample. The rescaling procedure is applied to all sectors but three, namely Wholesale; Retail, including Motor Vehicles; and Fuel. For these three sectors, the average labor share is low in the French data compared to the WIOD. This comes from the treatment of merchandise, which we categorize as intermediates while WIOD does not. Our approach is consistent with the model in the case of France, where we

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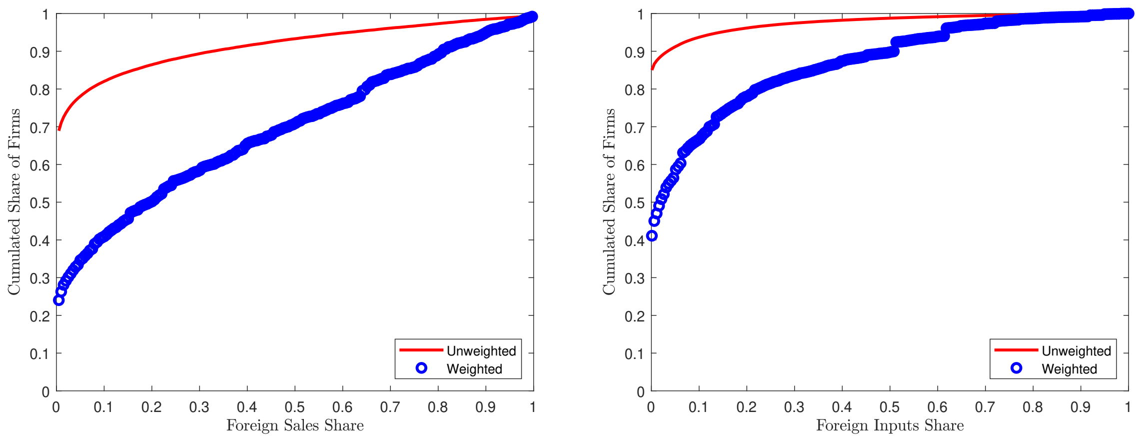
Figure A1. Distributions of Export and Imported Input Use Intensities Across French Firms



(a) Export Intensity **(b)** Imported Input Use Intensity

Notes: The left panel plots the cumulative distribution of firms according to their degree of exporting intensity, defined by the share of their sales going to foreign markets. The right panel plots the cumulative distribution of firms according to their share of inputs coming from other countries in total input spending. The solid (red) lines correspond to the unweighted distributions and the (blue) circles to the weighted distributions, where firms' weights are defined according to their share in the aggregate value added. The left panel is restricted to firms in the tradable sectors. Source: French customs and balance sheet data for 2005.

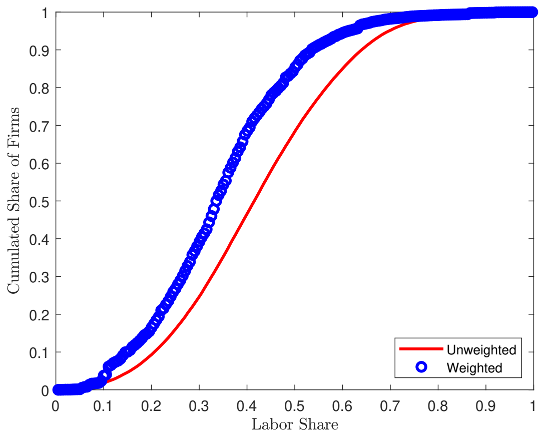
Figure A2 displays the cumulative distribution of labor shares, distinguishing between tradable and non-tradable sectors. The solid (red) line correspond to the unweighted distributions and the (blue) circles to the weighted ones. These distributions show a high degree of heterogeneity across firms, both within and across broad sectors. In traded good sectors, large firms tend to be less labor intensive, although the pattern is not systematic in all individual sectors and is not very strong. On the contrary, large firms in non-traded good sectors are often more labor-intensive than smaller ones.[2]

Total input usage at the firm level equals one minus the labor share (in our setting "labor" stands for the composite of primary factors). We further disaggregate total input usage across sectors and source countries using the information on imports, by product. This allows us to recover the $\pi^{M}_{f,mn,ij}$ coefficients for $n = France$. While in principle straightforward, calibrating these parameters entails two key difficulties: i) it requires the use of two sources of firm-level data,

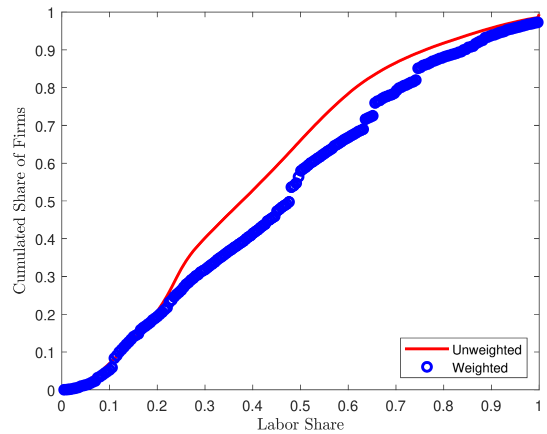
assume that consumers never interact directly with foreign firms. From that point of view, all merchandise imported from abroad is used as inputs by a French firm which ultimately sells to the final consumer. Because this is all the more important for retailing and wholesaling activities, we decided to keep the distribution of measured $\pi^{l}_{f,n,j}$ unchanged in these sectors.

[2] In the tradable sectors, the correlation between the firm's labor share and its size is on average mildly negative at -0.05, ranging between -0.25 (Coke, Refined Petroleum and Nuclear Fuel) and 0.09 (Wood Products). In the non-tradable sectors, it is on average positive but quite close to zero at 0.02, ranging from -0.9 in Water Transport to 0.12 in Real Estate Activities.

Figure A2. Distribution of Labor Shares Across French Firms



(a) Tradable Sectors



(b) Non-tradable Sectors

Notes: This figure plots the cumulative distributions of firm-level labor shares ($\pi^{l}_{f,n,j}$), in tradable and in non-tradable sectors. The solid (red) lines correspond to the unweighted distribution and the (blue) circles to the weighted distribution, where firms' weights are defined according to their share in aggregate value added. Calculated from French balance-sheet data together with the WIOD information on sectoral labor shares, for 2005.

which raises concerns regarding comparability; and ii) not all of these coefficients can be recovered from the firm-level data. In particular, we do not have detailed information on inputs purchased domestically and thus need to infer their sectoral breakdown using (more aggregated) information from WIOD. We proceed as follows.

For each sector i among the subset of tradable sectors and each source country $m \neq n$, we first compute the share of bilateral imports of goods produced by country m, sector i in the firm's total input expenses.[3] Since this ratio uses data collected from two databases, the overall import share obtained from the summation of these $\pi^{M}_{f,mn,ij}$ coefficients over all tradable sectors and foreign countries is larger than one in some cases (for less than 1.5% of firms). Whenever this happens, the import share is winsorized to one and the bilateral sectoral coefficients rescaled accordingly.

Beyond comparability issues between the two firm-level sources, the introduction of these firm-level import shares into the broader multi-country model also means we must ensure consistency with the sectoral coefficients in the global data. As we did with the labor shares, this implies rescaling the overall distribution of firm-level coefficients to the mean observed in the WIOD data:

$$\pi^{M}_{f,mn,ij} = \tilde{\pi}^{M}_{f,mn,ij} \frac{\pi^{M}_{mn,ij}}{\tilde{\pi}^{M}_{mn,ij}},$$

[3]This requires the conversion of product-level import data expressed in the highly disaggregated *Harmonized System* into broader sectoral categories. Since the customs data do not allow us to distinguish between the import of intermediates and merchandise (goods that are not further processed before being sold by the firm), we measure the firm's input expenses accordingly as the sum of raw materials *and* merchandise purchases (taking into account changes in inventories). See Blaum et al. (2018) for a similar treatment of the data.

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where $\pi^M_{f,mn,ij}$ and $\tilde{\pi}^M_{f,mn,ij}$ denote the rescaled and original firm-level coefficients, respectively, $\pi^M_{mn,ij}$ is the sectoral counterpart measured with the WIOD data, and $\tilde{\pi}^M_{mn,ij}$ is the weighted average of the firm-level original coefficients, where each firm is weighted according to its share $\omega^M_{f,n,j}$ in sectoral input purchases: $\tilde{\pi}^M_{mn,ij} = \sum_{f\in(n,j)} \omega^M_{f,n,j}\pi^M_{f,mn,ij}$. The normalization preserves as much heterogeneity across firms as possible, while avoiding overestimating the international transmission of shocks through foreign input purchases via an exaggeration of the degree to which French firms actually rely on foreign inputs. From that point of view, our calibration is conservative.

By definition, the remaining input purchases, those not sourced abroad, include tradable goods purchased in France and all expenses on non-tradable inputs. While we do not have any information on how these domestic expenses are spread across sectors, we can recover the firm-level share of individual input purchases as $\sum_i \pi^M_{f,nn,ij} = 1 - \sum_{m\neq n}\sum_{i\in T}\pi^M_{f,mn,ij}$. This domestic input share is then assigned to domestic input sectors using information in the WIOD:[4]

$$\pi^M_{f,nn,ij} = \frac{\pi^M_{nn,ij}}{\sum_i \pi^M_{nn,ij}} \times \sum_i \pi^M_{f,nn,ij}.$$

We have tested an alternative calibration strategy in which the input coefficients for non-traded sectors are all set exactly to their values in the WIOD. The remaining (homogeneous) share in input purchases is then spread across tradable sectors and countries using the bilateral import shares available at the firm level. The residual which corresponds to tradable inputs purchased domestically is spread across sectors using the WIOD coefficients. Note that this strategy tends to underestimate the share of tradable goods that are purchased domestically, i.e., it overestimates the participation of French firms to foreign input markets. For this reason, we have chosen to use the more conservative strategy described above as our benchmark.

A.2 Other Outcome Variables, Controls, and Trade Participation

Columns 1, 2, and 3 of Table A2 contain results for: (i) the labor costs, (ii) the materials spending, and (iii) the firm imported inputs. Just like total value added, the components of labor costs and materials spending are more sensitive to foreign GDP for larger firms. For real imported inputs the coefficient is positive but not statistically significant. At the same time, however, the sample size falls by more than four-fifths. These firms are already larger than the rest, and selected precisely on their importing status. Evidently, within this subsample the variation in size is not informative about the differential sensitivity to foreign shocks, perhaps because all importing firms are about equally sensitive to foreign GDP growth.

[4] Our definition of non-tradable (NT) sectors is somewhat unconventional since we de facto exclude from the tradable sector all services that are potentially traded but that we do not observe in the customs data. As a consequence, some of our NT sectors might display strictly positive foreign input shares in WIOD, i.e. $\pi^M_{mn,ij} \neq 0$ for $j \in NT$. We adjust the WIOD data to make them consistent with our definition of non-tradable sectors by allocating all purchases from a NT sector to the same French sector, i.e.: $\pi^M_{nn,ij} = \sum_m \pi^M_{mn,ij}$ and $\pi^M_{mn,ij} = 0, \forall i \in NT$. We apply the same adjustment to the other countries in the sample, to ensure comparability.

Table A3 performs a set of exercises that includes firm participation in international trade. Here the goal of the exercise is more subtle. Our model rationalizes the size-sensitivity to foreign shocks relationship through differential participation in import and export markets. Thus, to be consistent with the theory, we would like to see that the relationship between firm size and the sensitivity to foreign GDP growth becomes substantially attenuated once we control for trade linkages.

We pursue two strategies. In the first, we use im/exporting status as a control variable, by including dummies for im/exporting status, and their interactions with foreign GDP growth alongside our regressor of interest, which is the firm size-foreign GDP growth interaction. Column 1 of Table A3 reports the results. As hoped, the coefficient on the size-foreign GDP interaction falls by 40% in magnitude, and its significance level drops to 10%.

Because the regression in equation (6) is run at the firm-time level, it is not an ideal setting to explore the impact of importing and exporting on the comovement of firms with foreign countries, because both firm imports and exports are source/destination specific, and regression (6) does not include the source/destination dimension. The overall im/exporting status may not be a good control because different firms trade with different countries, and they should be more sensitive to GDP growth primarily of the countries with which they actually trade. To improve the quality of these controls, we construct alternative measures of sensitivity of im/exporters to foreign GDP growth, by picking out the GDP growth of only countries with which a firm trades:

$$d\ln Y^{IM}_{f,W,t} = \sum_m IM_{f,m,j,t-1} d\ln Y_{m,t}, \tag{A.1}$$

where $d\ln Y_{m,t}$ is GDP growth in country m, and $IM_{f,m,j,t-1}$ is the dummy that takes a value of 1 if firm f imports from country m in the initial period. We define the exporter-specific foreign growth in the same way:

$$d\ln Y^{EX}_{f,W,t} = \sum_m EX_{f,m,j,t-1} d\ln Y_{m,t}, \tag{A.2}$$

where $EX_{f,m,j,t-1}$ is a dummy for whether firm f exported to m in the initial period.

Column 2 of Table A3 reports the results. The original size-foreign GDP growth interaction coefficient falls further and becomes insignificant. This approach also leads to more sensible results when it comes to im/exporting status and sensitivity to foreign growth. The coefficients on both $d\ln Y^{IM}_{f,W,t}$ and $d\ln Y^{EX}_{f,W,t}$ are strongly significant and positive.

An alternative approach is to estimate separate size-foreign growth interaction coefficients by trade status. That is we run:

$$d\ln Y_{f,n,j,t} = \beta_0 d\ln Y_{W,t} + \sum_{\substack{S=DOM,\\ TRADE}} \beta_{1,S} \mathbb{1}_S \times \ln Y_{f,n,j,t-1} \times d\ln Y_{W,t} + \beta_2 \ln Y_{f,n,j,t-1} + \boldsymbol{\delta} + \epsilon_{f,t}. \tag{A.3}$$

In this specification, firms are split into 2 mutually exclusive categories: purely domestic (neither importer nor exporter), vs. trading. The specification (A.3) then allows these categories to have

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different sensitivities to foreign GDP growth. Column 3 of Table A3 reports the results. The most important finding is that purely domestic firms do not experience significantly greater sensitivity to foreign GDP growth if they are larger. The sensitivity to firm value added growth to foreign GDP growth is size-dependent only for firms that trade.

All in all, these results support our modeling approach. The baseline specification focuses on the differential sensitivity to foreign GDP growth by firm size because of parsimony and a tight connection to the notion of the granular residual, which is the covariance between firm size and firm-level responses to foreign shocks. Focusing on size provides an especially parsimonious way to summarize all the complexity in the importing and exporting links across firms, sources, and destinations, and connect our results directly to the granularity literature.[5]

In the model, this regularity is captured by the differences in the participation in international trade across firms of different sizes. Controlling for importing and exporting reduces the size-foreign GDP growth interaction coefficient substantially, and renders it insignificant. Furthermore, the greater sensitivity to foreign GDP growth is not observed for purely domestic firms. Thus, these results support our modeling approach in which international trade is the reason why larger firms are more sensitive to foreign shocks.

Next, we include the firm's multinational status and its interaction with world GDP growth as controls. To do this, we merge the *Liaisons Financières* (LiFi) survey into our data. This survey contains information on which firms in France have foreign affiliates, and which French firms are affiliates of foreign companies.[6] The information on multinational status is the only dimension of international financial integration at the firm level available to us, but it is an important one (see, e.g., Desai et al., 2004). Column 4 of Table A2 reports the results of controlling for a multinational dummy ($MNE_{f,t-1}$) and an interaction of the multinational dummy with foreign GDP growth. Foreign multinationals tend to be less responsive to foreign GDP growth than non-multinationals, though the coefficient is tiny in magnitude. However, even controlling for this effect, the size-growth interaction remains positive and significant, with the same magnitude as the baseline.[7]

Just like with goods trade above, multinational firms are only engaged with a subset of countries. That is, a French multinational company typically has affiliates in a limited number of foreign countries, and a French affiliate of a foreign multinational has its headquarters in a particular foreign country. Thus, we refine the multinational-foreign growth indicators in a way similar to what we did for trade indicators. Namely, for the French headquarters with affiliates abroad, we

[5] Since not all firms trade with all countries, an explicitly bilateral approach in which an observation is comovement between firm f and country m, is a better way to document that trade linkages create comovement. We pursue a full treatment of the bilateral approach in our 2018 paper (di Giovanni et al., 2018). Indeed, the finding that im/exporting firms are more correlated with foreign GDP is not new. These is an additional reason to focus on size as a single summary firm characteristic in the current paper.

[6] di Giovanni et al. (2018) describes these data in detail.

[7] We obtain a similar coefficient and significance level if we simply drop multinationals from the estimation sample.

construct the growth rate of countries where the firm has affiliates:

$$d\ln Y^{AF}_{f,W,t} = \sum_m AF_{f,m,j,t-1} d\ln Y_{m,t}, \tag{A.4}$$

where $AF_{f,m,j,t-1}$ is the dummy that takes a value of 1 if firm f has an affiliate in m. For French affiliates of foreign companies, we use the growth rate of the country in which the parent company is headquartered, $d\ln Y_{f,HQ,t}$. The results are in column 5 of Table A2. After this refinement, both the size and the significance of the size interaction are still essentially at their baseline values and significance levels.

Finally, we control for the real exchange rate. In principle, international relative prices are a conduit for international shock transmission, and in any general equilibrium model will be jointly determined with quantities. In practice, the exchange rates are disconnected from fundamentals (Itskhoki and Mukhin, 2021), and thus in the data might have a separate impact from world real GDP. Note that the main effect of exchange rate changes is absorbed in the fixed effects. However, we can still estimate the coefficient on the interaction of the exchange rate change with firm size. We obtain the Real Effective Exchange Rate series from the Bank of International Settlements via the Saint Louis Fed's FRED database, and compute the annual log change.[8] Column 4 of Table A3 presents the results. The main coefficient of interest on the world real GDP growth interaction actually becomes larger and remains highly significant. When it comes to the exchange rate, as noted above we cannot estimate the main effect of the exchange rate on firm value added growth. The differential effect turns out to be that larger firms benefit relatively more/suffer relatively less from a French appreciation. This is sensible, as larger firms are typically naturally hedged against exchange rate changes because they simultaneously import and export (e.g. Amiti et al., 2014). Smaller, purely domestic firms would be unambiguously negatively affected by a French appreciation due to greater competition from foreign goods. Larger firms are much more likely to import (the now cheaper) intermediate inputs, partly insulating them from greater foreign competition.

[8]The REER variable is defined in such a way that an increase in the index is an appreciation. The underlying REER data are monthly. Whether we take log differences vs. regular growth rates, or whether we use December-to-December vs. annual average-to-annual average changes turns out to be immaterial. Though we use the REER that takes into account movements in domestic price levels and is weighted by trade shares, in practice the REER has a 91% correlation with the Euro-USD exchange rate at the annual frequency, but is 3.5 times less volatile than the Euro-USD nominal rate. These patterns reflect the fact that while most of France's trade is with the Euro area, the variation in the REER comes from the movements in the nominal exchange rates with partners outside of the Euro area.

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Table A2. Sensitivity to Foreign GDP Growth by Firm Size: Other Outcomes and Multinational Status

Dep. Var.:	(1) *Labor costs*	(2) *Input costs*	(3) *Imported input costs*	(4) *Value added*	(5) *Value added*
$\ln Y_{f,m,j,t-1} \times d \ln Y_{W,t}$	0.095^a	0.352^a	0.406	0.108^a	0.108^a
	(0.021)	(0.032)	(0.621)	(0.030)	(0.030)
$\ln Y_{f,m,j,t-1}$	-0.006^a	-0.015^a	-0.076^a	-0.021^a	-0.021^a
	(0.001)	(0.001)	(0.019)	(0.001)	(0.001)
$MNE_{f,t-1}$				0.089^a	
				(0.011)	
$MNE_{f,t-1} \times d \ln Y_{W,t}$				-0.005^a	
				(0.001)	
$AF_{f,t-1}$					0.050^a
					(0.005)
$HQ_{f,t-1}$					0.049^a
					(0.002)
$d \ln Y^{AF}_{f,W,t}$					-0.594^a
					(0.065)
$d \ln Y^{HQ}_{f,W,t}$					0.504^a
					(0.081)
Observations	1,491,961	1,521,905	274,167	1,518,264	1,518,264
# years	11	11	11	11	11
# firms	136,478	138,355	39,974	138,024	138,024
Adjusted R^2	0.010	0.016	0.007	0.020	0.020
Fixed Effects	Sector×Year	Sector×Year	Sector×Year	Sector×Year	Sector×Year

Notes: Columns 1, 2, and 3 report the estimates of equation (6), with log changes in labor costs, materials costs, and imported input costs as dependent variables. Column 4 controls for the firm's multinational status, with the dummy variable $MNE_{f,t-1}$ taking on the values of 1 if the firm is part of either a French of a foreign multinational. Column 5 separates French headquarters with affiliates abroad ($AF_{f,t-1}$), and firms that are affiliates of a foreign firm ($HQ_{f,t-1}$), as well as the the differential effect of foreign growth by host-specific multinational status as in (A.4). $d \ln Y_{f,HQ,t}$ is the GDP growth rate of the country in which the parent of firm f is headquartered. Standard errors clustered at the firm level in parentheses with ^{a}, ^{b} and ^{c} denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

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Table A3. Sensitivity to Foreign GDP Growth by Firm Size and Trade Status

Dep. Var.: $d\ln Y_{f,m,j,t}$	(1)	(2)	(3)	(4)
$\ln Y_{f,m,j,t-1} \times d\ln Y_{W,t}$	0.060^{c}	0.049		0.445^{a}
	(0.034)	(0.030)		(0.035)
$IM_{f,t-1} \times d\ln Y_{W,t}$	0.040			
	(0.104)			
$EX_{f,t-1} \times d\ln Y_{W,t}$	0.522^{a}			
	(0.103)			
$d\ln Y^{IM}_{f,W,t}$		0.059^{a}		
		(0.005)		
$d\ln Y^{EX}_{f,W,t}$		0.032^{a}		
		(0.003)		
$\ln Y_{f,m,j,t-1}$	-0.023^{a}	-0.024^{a}		-0.288^{a}
	(0.001)	(0.001)		(0.001)
$IM_{f,t-1}$	0.027^{a}	0.022^{a}		
	(0.003)	(0.001)		
$EX_{f,t-1}$	0.005^{c}	0.017^{a}		
	(0.003)	(0.001)		
$DOM_{f,t-1} \times \ln Y_{f,m,j,t-1} \times d\ln Y_{W,t}$			0.045	
			(0.035)	
$TRADE_{f,t-1} \times \ln Y_{f,m,j,t-1} \times d\ln Y_{W,t}$			0.106^{a}	
			(0.030)	
$DOM_{f,t-1} \times \ln Y_{f,m,j,t-1}$			-0.021^{a}	
			(0.001)	
$TRADE_{f,t-1} \times \ln Y_{f,m,j,t-1}$			-0.023^{a}	
			(0.001)	
$DOM_{f,t-1}$			-0.033^{a}	
			(0.003)	
$\ln Y_{f,n,j,t-1} \times d\ln REER_{FRA,t}$				0.156^{a}
				(0.008)
Observations	1,518,264	1,518,264	1,518,264	1,518,264
# years	11	11	11	11
# firms	138,024	138,024	138,024	138,024
Adjusted R^2	0.022	0.022	0.021	0.021
Fixed Effects	Sector×Year	Sector×Year	Sector×Year	Sector×Year

Notes: Columns 1 and 2 report the estimates of equation (6), controlling for the differential effect of foreign growth on by import and export status $IM_{f,t-1}$ and $EX_{f,t-1}$, and source/destination-specific trade status as in (A.1) and (A.2). Columns 3 reports the results of estimating equation (A.3). Column 4 reports the results of estimating equation (6), constraining the sample to non-importing and non-exporting firms. Standard errors clustered at the firm level in parentheses with ^{a}, ^{b} and ^{c} denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

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Appendix B Theory and Quantification

B.1 The GDP Deflator Construction in the Model

This Appendix describes how we replicate the procedures used by the system of national accounts to compute changes in real GDP and the GDP deflator. The GDP deflator is an implicit deflator that is defined as the ratio of nominal and real GDP changes. In turn, the real GDP is computed using the "double deflation" method that records output net of inputs when both are evaluated at base prices. Specifically, define real GDP, evaluated at base prices (prices at -1) by:

$$Y_n = \sum_{j=1}^{\mathcal{J}} \left(P_{n,j,-1} Q_{n,j} - P^M_{n,j,-1} M_{n,j} \right),$$

where $Q_{n,j}$ is the gross physical output in sector j, $M_{n,j}$ is the physical use of inputs in the sector, $P_{n,j,-1}$ is the gross output base price, and $P^M_{n,j,-1}$ is the base price of inputs in that sector.

Denote by a "hat" a gross proportional change in a variable relative to its base value: $\widehat{x} \equiv x/x_{-1}$. The gross change in real GDP is then:

$$\widehat{Y}_n = \sum_{j=1}^{\mathcal{J}} \omega^D_{n,j,-1} \left(\widehat{Q}_{n,j} - \pi^M_{n,j,-1} \widehat{M}_{n,j} \right), \tag{B.1}$$

where $\omega^D_{n,j,-1} \equiv \frac{P_{n,j,-1} Q_{n,j,-1}}{Y_{n,-1}}$ is the base period Domar weight of sector j, that is, the ratio of the sector's gross sales to aggregate value added, and $\pi^M_{n,j,-1}$ is the base period sector-level share of input spending in gross output. Since $\omega^D_{n,j,-1}$ and $\pi^M_{n,j,-1}$ are both nominal beginning-of-period values, they are easily constructable from data.

To measure changes in physical quantities $\widehat{Q}_{n,j}$ and $\widehat{M}_{n,j}$, in practice national statistical agencies measure sectoral nominal gross sales and PPIs, and deflate the gross sales changes by PPI changes. That is, the pieces of data at the disposal of the statistical agencies are: nominal output in a sector, call it $P_{n,j} Q_{n,j}$, and a change in PPI, call it $\widehat{P}_{n,j}$. Then:

$$\widehat{Q}_{n,j} = \frac{1}{\widehat{P}_{n,j}} \times \frac{P_{n,j} Q_{n,j}}{P_{n,j,-1} Q_{n,j,-1}}.$$

For inputs, the mechanics are the same, but we have to know the change in the input price deflator in every sector, call it $\widehat{P}^M_{n,j}$. Then:

$$\widehat{M}_{n,j} = \frac{1}{\widehat{P}^M_{n,j}} \times \frac{P^M_{n,j} M_{n,j}}{P^M_{n,j,-1} M_{n,j,-1}}.$$

For the implementation inside our model, it is trivial to compute the sectoral nominal output and

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input spending growth relative to pre-shock values:

$$\frac{P_{n,j}Q_{n,j}}{P_{n,j,-1}Q_{n,j,-1}} = \frac{\sum_m \sum_{f\in\Omega_{nm,j}} X_{f,nm,j}}{\sum_m \sum_{f\in\Omega_{nm,j}} X_{f,nm,j,-1}}$$

$$\frac{P^M_{n,j}M_{n,j}}{P^M_{n,j,-1}M_{n,j,-1}} = \frac{\sum_m \sum_{f\in\Omega_{nm,j}} \left(1-\pi^l_{f,n,j}\right) X_{f,nm,j}}{\sum_m \sum_{f\in\Omega_{nm,j}} \left(1-\pi^l_{f,n,j,-1}\right) X_{f,nm,j,-1}}.$$

For price indices, in best practice of the statistical agencies, $\widehat{P}_{n,j}$ is just the PPI change. There is some heterogeneity across countries in whether the PPI includes export prices or not. For us, PPI will include exports, and will be computed as

$$\widehat{P}_{n,j} = \sum_m \sum_{f\in\Omega_{nm,j}} \omega^j_{f,nm,j,-1}\widehat{p}_{f,nm,j}, \tag{B.2}$$

where $\omega^j_{f,nm,j,-1} \equiv \frac{X_{f,nm,j,-1}}{\sum_m \sum_{f\in\Omega_{nm,j}} X_{f,nm,j,-1}}$ is the gross output weight of the firm's sales to m *in sector* j *sales*. Note that this is more comprehensive than what is actually done in practice, as the PPI is a survey that catches the minority of firms, and thus implementing (B.2) amounts to using more data than the statistical agencies do.

To construct the input price deflator $\widehat{P}^M_{n,j}$, the statistical agencies use the PPI and the IO tables. We mimic this procedure by computing the input-share weighted change in input prices, where we use the PPI for the domestic inputs, and the foreign sectoral price changes for foreign inputs. The important thing is that we carry this out at the sector level, without using any firm-level information:

$$\widehat{P}^M_{n,j} = \sum_i \sum_k \pi^M_{kn,ij,-1}\widehat{P}_{k,i}.$$

The $\pi^M_{kn,ij,-1}$'s are the input shares coming from the WIOD. For the domestic components of the right-hand side of this expression, the $\widehat{P}_{k,i}$ are just the PPI's from (B.2). For the foreign components, we assume that the foreign import prices are measured correctly, and use the import price indices from a particular country and sector, called $\widehat{P}_{kn,j}$ in the main text.

Now we have all the ingredients to compute the real GDP change (B.1). Since the GDP deflator is defined implicitly as the ratio between the nominal and real GDP change, we also need to compute the nominal GDP change. The nominal GDP change is a weighted sum of all firms' nominal value added changes. In particular, in our framework nominal value added associated with firm f's sales to market m is a constant fraction of its sales there:

$$Y^{NOM}_{f,nm,j} = \frac{1+\pi^l_{f,n,j}(\rho-1)}{\rho}X_{f,nm,j},$$

and thus total firm value added is given by:

$$Y^{NOM}_{f,n,j} = \frac{1+\pi^l_{f,n,j}(\rho-1)}{\rho}\sum_m X_{f,nm,j}.$$

Nominal GDP is simply the sum over all firm-level value added, as in (1). The change in GDP is:

$$\widehat{Y}_n^{NOM} = \sum_f \sum_m \omega_{f,n,j,-1} s_{f,nm,j,-1} \widehat{X}_{f,nm,j}, \tag{B.3}$$

where, as in Section 2, $\omega_{f,n,j,-1}$ is the pre-shock share of firm f's value added in total GDP, and $s_{f,nm,j,-1}$ is the pre-shock share of sales to m in firm f's total gross sales.

Finally, the GDP deflator is defined implicitly as the ratio of nominal and real GDP:

$$\widehat{P}_n^{GDP} = \frac{\widehat{Y}_n^{NOM}}{\widehat{Y}_n}.$$

B.2 A Shock Formulation of the Model

To perform counterfactuals that simulate the impact of foreign shocks on domestic firms and the aggregate economy, we follow the approach of Dekle et al. (2008) and express the equilibrium conditions in terms of gross changes $\widehat{x} = x/x_{-1}$ in endogenous variables, to be solved for as a function of shocks expressed in gross changes, and the pre-shock ("−1") observables. Starting with (10), we write it as a function of observed expenditure shares:

$$\begin{aligned} X_{mn,j} = \pi^c_{mn,j}\pi^c_{n,j}\left[w_n \left(\frac{1}{\psi_0}\frac{w_n}{P_n}\right)^{\frac{1}{\psi-1}} \overline{L}_n + \Pi_n + D_n \right] \\ + \sum_i \frac{\rho-1}{\rho} \sum_{f\in i} (1-\pi^l_{f,n,i})\pi^M_{f,mn,ji} \sum_k \pi_{f,nk,i} X_{nk,i}, \end{aligned} \tag{B.4}$$

where $\pi^c_{mn,j}$ is the share of final consumption spending on goods from m in the total consumption spending on goods in sector j, country n, $\pi^c_{n,j} = \vartheta_{n,j}$ is simply the share of sector j in total final consumption spending, and $\pi_{f,nk,i}$ is the share of firm f in the total exports from country n to country k in sector i. All of these π's are observable when n = France. $\pi^c_{mn,j}$ and $\pi^c_{n,j}$ are observable in WIOD. $\pi_{f,nk,i}$ when neither n nor k are France is not observable, so would require an assumption on which firms use imported intermediates. Since we do not have firm-level information on other countries, we assume that in those countries there is a representative firm in each sector. Writing out the shares:

$$\begin{aligned} \pi^c_{n,j} &= \vartheta_{n,j}, \\ \pi^c_{mn,j} &= \frac{\mu_{mn,j} P^{1-\sigma}_{mn,j}}{\sum_k \mu_{kn,j} P^{1-\sigma}_{kn,j}}, \\ \pi_{f,nk,i} &= \frac{\xi_{f,nk,i}\left(\frac{\rho}{\rho-1}\frac{\tau_{nk,i} b_{f,n,i}}{a_f}\right)^{1-\rho}}{P^{1-\rho}_{nk,i}}. \end{aligned}$$

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Then, in proportional changes relative to pre-shock values, (B.4) can be written as:

$$\widehat{X}_{mn,j}X_{mn,j,-1} = \pi^c_{mn,j}\pi^c_{n,j}\left[\widehat{w}_n\left(\frac{\widehat{w}_n}{\widehat{P}_n}\right)^{\frac{1}{\bar{\psi}-1}} s^L_{n,-1} + \widehat{\Pi}_n s^{\Pi}_{n,-1} + \widehat{D}_n s^D_{n,-1}\right] P_{n,-1}C_{n,-1} + \sum_i \frac{\rho-1}{\rho}\sum_{f\in i}(1-\pi^l_{f,n,i})\pi^M_{f,mn,ji}\sum_k \pi_{f,nk,i}\widehat{X}_{nk,i}X_{nk,i,-1}, \tag{B.5}$$

where $s^L_{n,-1}$ is the pre-shock share of labor (more generally factor payments) in the total final consumption expenditure, and the same for $s^{\Pi}_{n,-1}$ and $s^D_{n,-1}$.

Equation (12) is expressed in changes as:

$$\sum_j\sum_{f\in j}\sum_k \frac{\rho-1}{\rho}\pi^l_{f,n,j,-1}\pi_{f,nk,j,-1}X_{nk,j,-1}\left[\widehat{\pi}^l_{f,n,j}\widehat{\pi}_{f,nk,j}\widehat{X}_{nk,j} - \widehat{w}_n^{\frac{\bar{\psi}}{\bar{\psi}-1}}\widehat{P}_n^{\frac{1}{1-\bar{\psi}}}\right] = 0. \tag{B.6}$$

The prices (11) are expressed in changes as:

$$\widehat{P}_{mn,j} = \left[\sum_{f\in\Omega_{mn,j}}\pi_{f,mn,j,-1}\widehat{\xi}_{f,mn,j}\left(\widehat{b}_{f,m,j}\widehat{a}_f^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{B.7}$$

$$\widehat{P}_{n,j} = \left[\sum_m \widehat{P}^{1-\sigma}_{mn,j}\pi^c_{mn,j,-1}\right]^{\frac{1}{1-\sigma}}, \tag{B.8}$$

$$\widehat{P}_n = \prod_j \widehat{P}^{\vartheta_{n,j}}_{n,j}. \tag{B.9}$$

Finally, the equations above require knowing post-shock π's. These can be expressed as:

$$\pi^c_{mn,j} = \frac{\widehat{P}^{1-\sigma}_{mn,j}\pi^c_{mn,j,-1}}{\sum_k \widehat{P}^{1-\sigma}_{kn,j}\pi^c_{kn,j,-1}}, \tag{B.10}$$

$$\pi_{f,nk,j} = \frac{\widehat{\xi}_{f,nk,j}\left(\widehat{b}_{f,n,j}\widehat{a}_f^{-1}\right)^{1-\rho}\pi_{f,nk,j,-1}}{\sum_{g\in\Omega_{nk,j}}\widehat{\xi}_{g,nk,j}\left(\widehat{b}_{g,n,j}\widehat{a}_g^{-1}\right)^{1-\rho}\pi_{g,nk,j,-1}}, \tag{B.11}$$

$$\widehat{b}_{f,m,j} = \left[\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi} + (1-\pi^l_{f,m,j,-1})\left(\widehat{P}^M_{f,m,j}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}, \tag{B.12}$$

$$\widehat{P}^M_{f,m,j} = \left[\sum_i\sum_k \pi^M_{f,km,ij,-1}\widehat{P}^{1-\eta}_{km,i}\right]^{\frac{1}{1-\eta}}, \tag{B.13}$$

$$\pi^l_{f,m,j} = \frac{\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi}}{\pi^l_{f,m,j,-1}\widehat{w}_m^{1-\phi} + (1-\pi^l_{f,m,j,-1})\left(\widehat{P}^M_{f,m,j}\right)^{1-\phi}}, \tag{B.14}$$

$$\pi^M_{f,km,ij} = \frac{\pi^M_{f,km,ij,-1}\widehat{P}^{1-\eta}_{km,i}}{\sum_i\sum_n \pi^M_{f,nm,ij,-1}\widehat{P}^{1-\eta}_{nm,i}}. \tag{B.15}$$

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B.2.1 Model Solution and Calibration

The model implementation involves solving equations (B.5)-(B.15). In particular, we solve for the following equilibrium variables:

1. Changes in trade values $\widehat{X}_{mn,j}$ $\forall m,n,j$;
2. Changes in wages $\widehat{w}_n$ $\forall n$;
3. Changes in the price indices $\widehat{P}_n$ $\forall n$, $\widehat{P}_{n,j}$ $\forall n,j$, $\widehat{P}_{mn,j}$ $\forall m,n,j$;
4. Post-shock trade shares $\pi^c_{mn,j}$ $\forall m,n,j$, $\pi_{f,nk,j}$ $\forall k,n,j,f$, $\pi^l_{f,n,j}$ $\forall n,j,f$, $\pi^M_{f,mn,ij}$ $\forall n,m,i,j,f$.

We further require several pre-shock data series, either at the firm or sector level. Specifically, we require information on:

1. Gross sales $X_{mn,j,-1}$ $\forall m,n,j$;
2. Final consumption shares within a sector across sources $\pi^c_{mn,j,-1}$ $\forall m,n,j$;
3. Firm-level within sector, within-destination trade shares $\pi_{f,nk,j,-1}$ $\forall k,n,j,f$;
4. Final consumption spending $P_{n,-1}C_{n,-1}$;
5. Shares of labor (factor) income, pure profits, and deficits in final consumption spending $s^L_{n,-1}$, $s^{\Pi}_{n,-1}$ and $s^D_{n,-1}$ $\forall n$;
6. Initial input shares $\pi^l_{f,n,j,-1}$ $\forall n,j,f$, $\pi^M_{f,mn,ij,-1}$ $\forall m,n,i,j,f$.

The construction of these variables and the relevant data sources are described in Appendix A. The solution of the model further requires setting a small number of parameter values. These are summarized in Table 2.

B.2.2 Satisfying Market Clearing

In order to proceed correctly with the hat algebra in each sector/country pair, in the pre-period the market clearing condition in levels must be satisfied:

$$X_{mn,j,-1} = \pi^c_{mn,j,-1}\pi^c_{n,j,-1}P_{n,-1}C_{n,-1} + \sum_i \frac{\rho-1}{\rho}\sum_{f\in i}(1-\pi^l_{f,n,i,-1})\pi^M_{f,mn,ji,-1}\sum_k \pi_{f,nk,i,-1}X_{nk,i,-1}. \tag{B.16}$$

In the data, this is unlikely to be the case. We therefore adopt the following approach: in each mn,j, trivially we can find a wedge $\zeta_{mn,j,-1}$ such that conditional on all the other data, (B.16) does hold with equality:

$$X_{mn,j,-1} = \pi^c_{mn,j,-1}\pi^c_{n,j,-1}P_{n,-1}C_{n,-1}+\sum_i \frac{\rho-1}{\rho}\sum_{f\in i}(1-\pi^l_{f,n,i,-1})\pi^M_{f,mn,ji,-1}\sum_k \pi_{f,nk,i,-1}X_{nk,i,-1}+\zeta_{mn,j,-1}.$$

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Then applying the hat algebra to this equation:

$$\begin{aligned}\widehat{X}_{mn,j}X_{mn,j,-1} = \pi^c_{mn,j}\pi^c_{n,j}\left[\widehat{w}_n\left(\frac{\widehat{w}_n}{\widehat{P}_n}\right)^{\frac{1}{\psi-1}}s^L_{n,-1} + \widehat{\Pi}_n s^{\Pi}_{n,-1} + \widehat{D}_n s^D_{n,-1}\right]P_{n,-1}C_{n,-1} \\ + \sum_i \frac{\rho-1}{\rho}\sum_{f\in i}(1-\pi^l_{f,n,i})\pi^M_{f,mn,ji}\sum_k \pi_{f,nk,i}\widehat{X}_{nk,i}X_{nk,i,-1} \\ + \widehat{\zeta}_{mn,j}\zeta_{mn,j,-1}.\end{aligned} \tag{B.17}$$

Next, we solve the entire model while feeding in a "shock" that eliminates this wedge, namely: $\widehat{\zeta}_{mn,j} = 0$. Finding the model solution will give the a set of $\widehat{X}_{mn,j}$'s that are required to arrive at a set of levels of $X_{mn,j}$ for which the market clearing condition is satisfied with equality for every mn, j. Then use these $X_{mn,j}$ as the starting (pre-shock) values for all the real counterfactuals we run. The antecedent of this approach is Costinot and Rodríguez-Clare (2014), who use a similar device to eliminate the trade deficits.

B.3 Simulating Actual Foreign GDP Growth

In any year in the data, there will be a vector of country-specific productivity shocks. Let $\epsilon_{f,m} \equiv d\ln Y^F_{f,n}/d\ln a_m$ denote the elasticity of value added of firm f to a productivity shock in country m. We obtain these elasticities for every firm in France and every partner country by simulating country-specific aggregate productivity shocks $d\ln a_m$ in the baseline model, and recording each firm's responses to it. Firm f's real value added growth rate following a vector of foreign shocks is

$$d\ln Y^F_{f,n} = \sum_m \epsilon_{f,m} d\ln a_m. \tag{B.18}$$

Then the change in French GDP due to a worldwide vector of foreign shocks is simply:

$$d\ln Y^F_n = \sum_f \omega_{f,n,-1} d\ln Y^F_{f,n}. \tag{B.19}$$

We implement (B.18)-(B.19) in two ways. The first approach feeds the aggregate TFP shocks from the Penn World Tables directly into (B.18) to compute each firm's response to those foreign TFP shocks. The second approach uses actual GDP growth rates. To compute the propagation of foreign GDP growth rates into France, we re-express (B.18) directly in terms of elasticities of French firms to foreign GDP. Specifically, instead of (B.18) we assume that firm growth rate following a country-specific shock is:

$$d\ln Y^F_{f,n} = \sum_m \widetilde{\epsilon}_{f,m} d\ln Y_m, \tag{B.20}$$

where $\widetilde{\epsilon}_{f,m} \equiv d\ln Y^F_{f,n}/d\ln Y_m$ is the elasticity of firm f's value added growth to country m's GDP, rather than the TFP shock directly. The $\widetilde{\epsilon}_{f,m}$'s can be computed by simulating a country-specific shock and tracking the response of both firm f and the foreign country's GDP. Equation (B.20) is then combined with (B.19) to compute French GDP growth. Once we simulate the firm and aggregate growth rates due to actual changes in foreign TFP and GDP for a sample of years, we can compute the average-granular residual decomposition (3).

Note that implementing (B.18)-(B.19)-(B.20) amounts to the first-order approach, where firm and aggregate responses are linear functions of the vector of foreign shocks. Huo et al. (2019) analyze the properties of the linear solution in a similar environment, and show that the first-order solution is very close to the exact one.

B.4 Proof of Proposition 1

Since all firms have the same production function, their initial labor shares $\pi^l_{n,i,-1}$ and input shares $\pi^M_{mn,ji,-1}$ are identical within a sector. All firms face the same effective intermediate input price change: $\widehat{P}^M_{f,m,j} = \widehat{P}^M_{m,j}\ \forall f$ (see (B.13)). Then, it is immediate from (B.14) and (B.15) that the post-shock labor and input shares $\pi^l_{n,i}$ and $\pi^M_{mn,ji}$ are also identical within a sector. The market clearing condition (B.5) becomes:

$$\widehat{X}_{mn,j}X_{mn,j,-1} = \pi^c_{mn,j}\pi^c_{n,j}\left[\widehat{w}_n\left(\frac{\widehat{w}_n}{\widehat{P}_n}\right)^{\frac{1}{\bar{\psi}-1}} s^L_{n,-1} + \widehat{\Pi}_n s^{\Pi}_{n,-1} + \widehat{D}_n s^D_{n,-1}\right] P_{n,-1}C_{n,-1}$$
$$+ \sum_i \frac{\rho-1}{\rho}(1-\pi^l_{n,i})\pi^M_{mn,ji}\sum_k \widehat{X}_{nk,i}X_{nk,i,-1},$$

and thus does not involve $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s or any other firm-level objects.

Since all firms face the same input bundle cost change: $\widehat{b}_{f,m,j} = \widehat{b}_{m,j}\ \forall f$ (see (B.12)), the $\pi_{f,nk,j}$ updating Equation (B.11) becomes:

$$\begin{aligned}
\pi_{f,nk,j} &= \frac{\widehat{\xi}_{f,nk,j}\left(\widehat{b}_{f,n,j}\widehat{a}_f^{-1}\right)^{1-\rho}\pi_{f,nk,j,-1}}{\sum_{g\in\Omega_{nk,j}}\widehat{\xi}_{g,nk,j}\left(\widehat{b}_{g,n,j}\widehat{a}_g^{-1}\right)^{1-\rho}\pi_{g,nk,j,-1}} \\
&= \frac{\pi_{f,nk,j,-1}}{\sum_{g\in\Omega_{nk,j}}\pi_{g,nk,j,-1}} \\
&= \pi_{f,nk,j,-1},
\end{aligned}$$

since taste and productivity shocks are not firm-specific, and the denominator sums to 1. Thus, sales shares are unchanged following a foreign shock: $\pi_{f,nk,j} = \pi_{f,nk,j,-1}\ \forall f,k$, or $\widehat{\pi}_{f,nk,j} = 1\ \forall f,k$.

When labor shares $\pi^l_{n,i,-1}$ do not differ across firms, the labor market condition (B.6) also does not require firm-level shares, and simplifies to:

$$\sum_j\sum_k \frac{\rho-1}{\rho}\pi^l_{n,j,-1}X_{nk,j,-1}\left[\widehat{\pi}^l_{n,j}\widehat{X}_{nk,j} - \widehat{w}_n^{\frac{\bar{\psi}}{\bar{\psi}-1}}\widehat{P}_n^{\frac{1}{1-\bar{\psi}}}\right] = 0,$$

which once again is independent of $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s.

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Finally, the price equation also has no $\pi_{f,nk,j}$ or $\pi_{f,nk,j,-1}$ terms if taste and productivity shocks are not firm-specific:

$$\begin{aligned}\widehat{P}_{mn,j} &= \left[\sum_{f\in\Omega_{mn,j}} \pi_{f,mn,j,-1}\widehat{b}_{m,j}^{1-\rho}\right]^{\frac{1}{1-\rho}} \\ &= \widehat{b}_{m,j}\end{aligned}$$

These equations define the equilibrium in changes, and thus $\widehat{X}_{mn,j}$'s and $\widehat{P}_{mn,j}$'s can be found without knowing the firm-level market shares $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s.

Since markups are constant, all the firm-specific prices change by the same proportional amount: $\widehat{p}_{f,mn,j} = \widehat{p}_{mn,j}\ \forall f$. Because $\widehat{\pi}_{f,mn,j} = 1\ \forall f, n$, all nominal sales changes are the same across firms within a sector: $\widehat{X}_{f,mn,j} = \widehat{X}_{mn,j}$. Therefore, none of the steps in constructing real GDP in Appendix B.1 require firm-level sales shares. □

B.5 $2 \times 2 \times 2$ Model Calibration

Top panel of Table A4 reports the input coefficients in the homogeneous firm model. In the homogeneous firm model, 24% of a Tradable sector firm's total costs (intermediates plus primary factors) are spent on foreign inputs, with the remaining 76% on domestic intermediates and labor. In the Non-Tradable sector, 8% of total costs go to pay for foreign inputs. These values correspond to the WIOD data when collapsed to 2 sectors and 2 countries, France and ROW.

The bottom panel of Table A4 reports the input coefficients in the final simulation. In the Tradable sector 47% of Firm 1's costs are spent on foreign inputs. Across simulations, we keep the sector-level share of spending on imported inputs constant in the Tradable sector at 24%. Thus, Firm 2's share of imported inputs is now 1% (recall that these firms have the same sales). The same reassignment of import shares occurs in the Non-Tradable sector.

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Table A4. Input Coefficients and Domar Weights in the $2 \times 2 \times 2$ Model

	Tradable		Non-Tradable	
	Firm 1	Firm 2	Firm 1	Firm 2
	Homogeneous Input Shares			
Share of inputs from:				
France	0.76	0.76	0.92	0.92
ROW	0.24	0.24	0.08	0.08
Domar weight	0.21	0.21	0.52	0.52
	Heterogeneous Input Shares			
Share of inputs from:				
France	0.53	0.99	0.86	0.99
ROW	0.47	0.01	0.14	0.01
Domar weight	0.21	0.21	0.52	0.52

Notes: This table reports the firm-specific input coefficients and Domar weights in the simplified $2 \times 2 \times 2$ model.

Table A5. Responses of French Real GDP to 10% Foreign Productivity and Demand Shocks, CPI Deflation

	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F
Shock:	**Productivity**			**Demand**		
Baseline	6.38	4.24	2.15	0.47	0.31	0.16
Share:		*0.664*	*0.336*		*0.649*	*0.351*
Homogeneous firms	7.34	7.31	0.03	0.53	0.51	0.02
Share:		*0.996*	*0.004*		*0.968*	*0.032*
	Sector-Level Decomposition					
	$d\ln Y^F$	$\mathcal{E}^F_{\mathcal{J}}$	$\Gamma^F_{\mathcal{J}}$	$d\ln Y^F$	$\mathcal{E}^F_{\mathcal{J}}$	$\Gamma^F_{\mathcal{J}}$
Baseline	6.38	5.72	0.66	0.47	0.72	-0.25
Share:		*0.897*	*0.103*		*1.525*	*-0.525*

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock (left panel) or a 10% foreign demand shock for French goods (right panel) in every other country in the world, in both the baseline model and the alternative model that suppresses firm heterogeneity. The table reports the decomposition of the the GDP change into the unweighted average and granular residual terms as in (3). The real GDP is obtained by deflating by CPI.

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Table A6. Robustness: GDP Responses to a Worldwide Productivity Shock in the Baseline vs. Homogeneous Models

Model:	**Baseline**			**Homogeneous firms**		
	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F
Main calibration	2.77	0.70	2.07	3.39	3.36	0.03
ρ:						
high: 5	1.73	-2.26	4.00	2.98	2.90	0.08
low:1.5	3.77	3.37	0.40	4.03	4.16	-0.13
Frisch:						
high: 2	14.43	11.02	3.42	16.21	16.14	0.06
low: 0.1	0.89	-0.78	1.68	1.26	1.24	0.02
η:						
high: 1.5	2.30	0.25	2.06	2.71	2.59	0.12
low: 0.5	3.40	1.28	2.12	4.30	4.37	-0.07
ϕ:						
high: 1.5	2.87	0.69	2.19	3.63	3.65	-0.02
low: 0.5	2.60	0.72	1.88	3.07	2.99	0.08
σ:						
high: 3	0.56	-1.23	1.79	0.92	0.76	0.15
low: 1.1	3.62	1.38	2.24	4.37	4.36	0.00
Flexible markups	2.89	0.80	2.10	3.39	3.36	0.03
Changing profits in final demand	3.20	1.12	2.09	3.77	3.72	0.06

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock in every other country in the world, and decomposes the total GDP change into the $\mathcal{E}^F$ and Γ^F terms. The left panel reports the results for the baseline model, and the right panel for the alternative model that suppresses firm heterogeneity, for alternative parameter values.

Table A7. Robustness: GDP Responses to a Worldwide Preference Shock in the Baseline vs. Homogeneous Models

Model:	**Baseline**			**Homogeneous firms**		
	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F	$d\ln Y^F$	$\mathcal{E}^F$	Γ^F
Main calibration	0.35	0.19	0.17	0.38	0.37	0.02
ρ:						
high: 5	0.29	0.09	0.21	0.34	0.35	-0.00
low:1.5	0.45	0.26	0.19	0.46	0.39	0.07
Frisch:						
high: 2	0.58	0.41	0.16	0.63	0.61	0.02
low: 0.1	0.29	0.13	0.17	0.31	0.30	0.02
η:						
high: 1.5	0.51	0.27	0.25	0.54	0.52	0.02
low: 0.5	0.16	0.08	0.08	0.18	0.17	0.01
ϕ:						
high: 1.5	0.36	0.20	0.16	0.39	0.37	0.01
low: 0.5	0.35	0.18	0.17	0.38	0.36	0.02
σ:						
high: 3	0.98	0.45	0.53	1.05	0.99	0.06
low: 1.1	0.11	0.06	0.05	0.12	0.12	0.00
Flexible markups	0.37	0.19	0.18	0.38	0.37	0.02
Changing profits in final demand	0.35	0.18	0.17	0.38	0.38	0.00

Notes: This table reports the change in French GDP, in percentage points, following a 10% foreign demand shock for French goods, and decomposes the total GDP change into the $\mathcal{E}^F$ and Γ^F terms. The left panel reports the results for the baseline model, and the right panel for the alternative model that suppresses firm heterogeneity, for alternative parameter values.

Table A8. Robustness: GDP Responses in the Baseline vs. Homogeneous Models

Shock:	**Productivity**			**Demand**		
	Baseline	Homogeneous	Ratio H/B	Baseline	Homogeneous	Ratio H/B
Main calibration	2.77	3.39	1.22	0.35	0.38	1.08
ρ:						
high: 5	1.73	2.98	1.72	0.29	0.34	1.18
low: 1.5	3.77	4.03	1.07	0.45	0.46	1.02
Frisch:						
high: 2	14.43	16.21	1.12	0.58	0.63	1.09
low: 0.1	0.89	1.26	1.41	0.29	0.31	1.07
η:						
high: 1.5	2.30	2.71	1.18	0.51	0.54	1.06
low: 0.5	3.40	4.30	1.27	0.16	0.18	1.11
ϕ:						
high: 1.5	2.87	3.63	1.26	0.36	0.39	1.08
low: 0.5	2.60	3.07	1.18	0.35	0.38	1.08
σ:						
high: 3	0.56	0.92	1.64	0.98	1.05	1.07
low: 1.1	3.61	4.37	1.21	0.11	0.12	1.06
Flexible markups	2.89	3.39	1.17	0.37	0.38	1.03
Changing profits in final demand	3.20	3.77	1.18	0.35	0.38	1.11

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock (left panel) or a 10% foreign demand shock for French goods (right panel) in every other country in the world, both in the baseline model and the alternative model that suppresses firm heterogeneity, for alternative parameter values.

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