Online Appendix for

Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy

Julian di Giovanni[⋆] Manuel García-Santana[⋄] Priit Jeenas[□]
Enrique Moral-Benito[∘] Josep Pijoan-Mas[△]

November 2, 2023

A Details on the data

A.1 Public procurement data

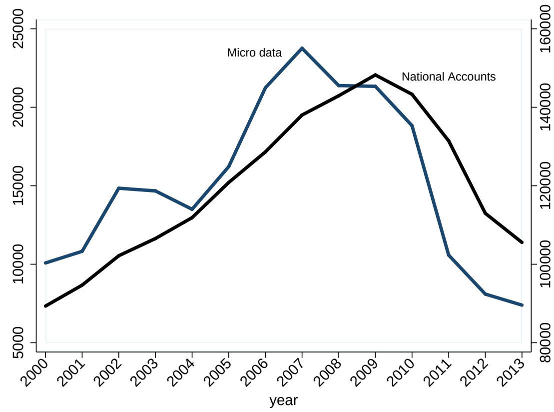
According to the System of National Accounts (SNA), "Government consumption expenditures and gross investment", i.e., G, measures the fraction of GDP, or final expenditures, that is accounted for by the government sector. Public procurement is defined in the *System of National Accounts (SNA)* as the sum of intermediate consumption (e.g., purchases of goods like medical consumables and services like accounting services), gross capital formation (e.g., building new roads), and social transfers in kind via market producers (e.g., medicines). Roughly speaking, one can think of public procurement as "government consumption expenditures and gross investment" (the G part of GDP) minus "compensation of employees" and "consumption of fixed capital." The size of public procurement varies across countries and over time. For OECD countries during 2007-2017, public procurement represented roughly 12% of GDP and 30% of G, on average.

Figure A.I shows the evolution of procurement value as measured with our micro data and compares it to the counterpart from national accounts. On average, our micro data accounts for around 13% of total government procurement as measured in Spanish national accounts. Our micro data reproduces well the cyclical aspect of public procurement expenditure, increasing during the boom and decreasing during the recession.

Main sample of projects published in BOE. According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[1] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity

[1] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

Figure A.I. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13. The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts.

that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin. We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides information on the type of contract (kind of good or service provided), the awarding institution, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped more than 150,000 projects over 2000-2013, which we assign to the month that the project was awarded. Of these, 130,633 projects have a value assigned to them that we were able to recover. The sum of all these projects totals around 220 billion euros. On average, our micro data account for around 13% of total public procurement as measured in National Accounts. Despite the level differences, our micro data are able to capture the overall evolution of public procurement over time, which increased from 9.9 to 13.8 percent between 2000 and 2009 and decreased from 13.8 to 10.0 percent between 2010 and 2013; see Figure A.I.

Small sample of projects with information on bidders. The BOE website does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period.

A.2 Balance sheet and credit data

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total fixed assets, and total assets. The final sample covers around 85-90% of non-financial firms for all size categories in terms of both turnover and number of employees.

Table A.I. Descriptive evidence from the final merged dataset, year 2006

	mean		**25th pctile**		**50th pctile**		**75th pctile**	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.42	10.95	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.56	12.75	16.00	3.00	45.00	6.00	155.0	12.00
Sales	8.96	1.19	1.14	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.20	0.00	0.01	0.00	0.03	0.00	0.10	0.00
Fixed Assets	3.80	0.85	0.21	0.03	0.82	0.14	3.58	0.50
Credit	2.51	0.57	0.11	0.03	0.48	0.08	2.32	0.30
Coll. Credit (share)	0.14	0.29	0.00	0.00	0.00	0.00	0.14	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract ($n = 2,411$) vs. the rest of the firms ($n = 406,261$). The variable *Employment* measures the number of full-time workers employed by the firm; the variable *Sales* is just firm's revenue measured in millions of euro; *Procurement/Sales* measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year; *Assets* measures the value of fixed assets; *Credit* measures the value of all firm's outstanding loans in millions of euro; *Coll. Credit (share)* is the share of *Credit* collateralized against firm's assets; *Def. Credit (share)* is the share of defaulted credit over total *Credit*; *age* measures the age of the firm. We winsorize the 1% tails of all variables.

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding bank debt appear in the CIR. In addition to the total amount of credit, CIR also contains information on whether or not a non-personal collateral ("Garantía real") was posted for a particular loan. These collaterals include assets like real estate, land, machinery, securities, deposits, and merchandise (i.e., hard collateral). With this information, we can hence assess whether a particular loan for a bank-firm pair was granted on the basis of tangible collateral.
Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as

follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[2] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding loans from CIR, we can infer whether the loan was granted or not.

B Heterogeneous effects of credit growth

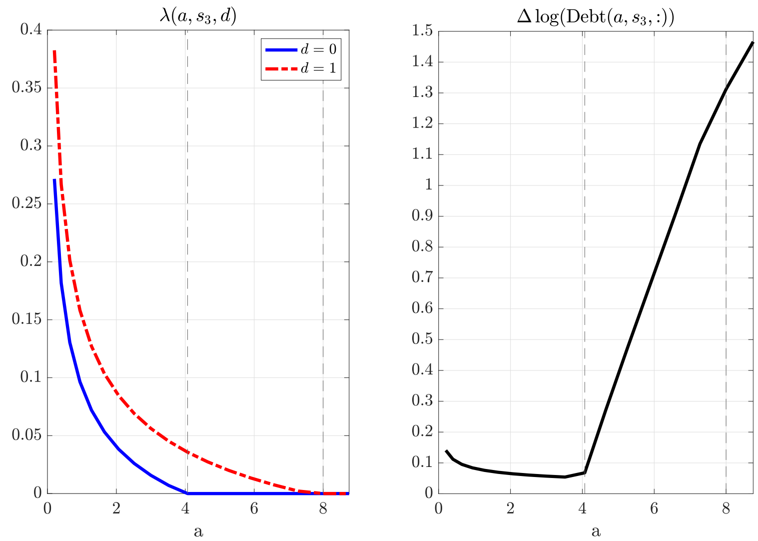
We investigate the heterogeneous effects of procurement on firms' credit growth. Before looking at the data, we first shed light on this relationship using our model's calibrated version. In panel (a) of Figure A.II, we show how the Lagrange multiplier associated with the financial constraint, $\lambda(s, a, d)$, changes across firms with different levels of net-worth, a.[3] We show this relationship for firms with procurement, i.e., $d = 1$, and without procurement, i.e., $d = 0$. As discussed in Section 4.6 of the main text, higher net-worth (a) firms are less constrained, and, conditional on a, procurement firms ($d = 1$) are more likely to be constrained because of their higher demand. The dashed lines divide the graph into three regions. The region to the left is one in which firms, with and without procurement, are financially constrained. In the middle region, only firms with procurement are constrained. In the region to the right, all firms are unconstrained.

In panel (b), we show the change in credit on impact ($h = 0$), of a firm becoming active in procurement ($d = 0 \rightarrow d = 1$), as a function of net worth a, i.e., the inverse of financial constraints all else equal. The main takeaway from this graph is that the model predicts a non-monotonic relationship between firms' financial constraints and the effect of procurement on credit. When the financial constraint is binding both before and after the procurement shock (left region), the model predicts that less financially constrained firms (higher a and hence lower λ) exhibit a smaller increase in credit when becoming active in procurement. However, the impact for firms in the other two groups (when transitioning from unconstrained to constrained or remaining unconstrained) increases with net worth. As a result, the model exhibits the u-shaped relationship between net credit growth and net worth.

In Section 6.2 we discuss the intuition for why unconstrained firms exhibit a larger increase in credit due to the procurement shock. The idea is that these firms, precisely because they are unconstrained, can expand freely by increasing credit. On the contrary, even if pro-

[2]The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

[3]We produce this graph fixing the level of firms' productivity at the middle point in our productivity grid, i.e., s_3. For other levels of s, the graph would simply be an identical, scaled, version of Figure A.II.



(a) Lagrange multiplier λ **(b)** change in log credit

Notes: Panel (a) shows the Lagrangian multiplier associated to the financial constraint, λ, for firms with different levels of net-worth in our model, both for firms with and without procurement ($d = 0$ and $d = 1$). The dotted lines organize the graph in three regions. In Panel (b) the effect on impact, i.e., $h = 0$, for different levels of firms' financial constraints as proxied by a (and implicitly given by λ).

Figure A.II. Het. effects of credit (calibrated model)

curement allows them to borrow more, constrained firms remain constrained, limiting the amount of credit they can raise after the demand shock implied by procurement.

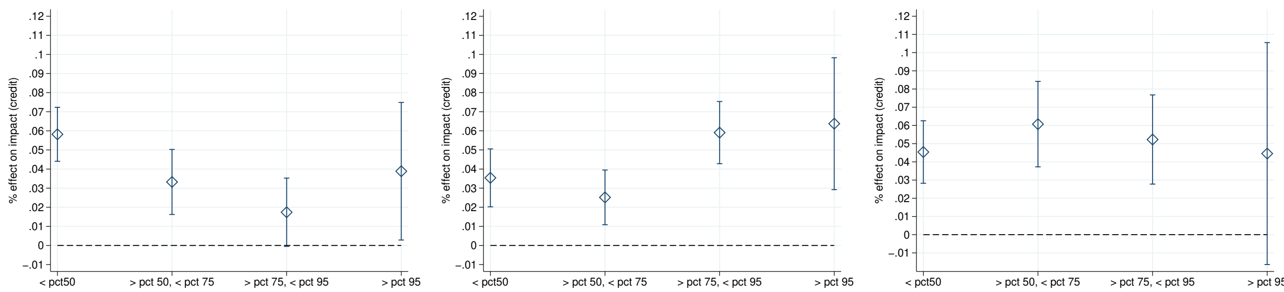
We next look at the heterogeneous effects of procurement on firms' credit in the data. Similarly to Section 3.1 in the main text, we show the impact effect, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). See Figure A.III. As in the model, the relationship between proxies for firms' financial constraints and the change in credit is non-monotonic. With fixed assets and leverage as proxies, the empirical evidence is reasonably consistent with the u-shaped relationship predicted by the model.

C Additional evidence

C.1 Effects on impact using quarterly data

To document a stronger identification of the effect of procurement on credit growth, we use quarterly data. We first regress firms' credit growth on a dummy variable for procurement:

$$\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it} \tag{C.1}$$



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

Figure A.III. Heterogeneous effects of procurement on credit

where the dependent variable $\Delta \log l_{it}$ is the annualized quarterly growth of credit (loans) of firm i between quarter $t-1$ and quarter t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$. The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in quarter t. We include the firm's lagged credit at $t-1$ to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels.[4] We further include a stringent set of fixed effects. In particular, we use firm×year fixed effects, α_{iy}, in order to capture firm-level characteristics that vary over time at the yearly (y) level. Importantly, these fixed effects help control for several factors that may otherwise bias the estimation. First, as they vary at the year level, they pick up the overall firm-level trend of credit growth and thus helps assuage the concern of any potential bias arising from differences in trends pre/post "treatment" by procurement events. Second, these fixed effects control for any firm-level variables that may change at annual level such as productivity or demand. We further include 4-digit sector×quarter effects, α_{st}, which control for both sector and macroeconomic conditions that vary over time. Thus, identification of the key parameter of interest, β_1, comes from the variation of a firm's credit growth across quarters within a year conditional on obtaining a procurement contract.

Table A.II, column (1), presents the results of this regression for the main sample. The estimate of β_1 is positive and significant at the one-percent level.[5] The estimated coefficient implies that winning a procurement contract in a quarter translates into an increase of credit growth of 5.5 percentage points annually.

[4] The estimation results without lagged credit are similar and are available upon request.

[5] We cluster standard errors at the firm-level in all regressions unless otherwise noted.

Table A.II. Credit Growth and Procurement

	All firms	Bidders only	
		First	Second
	(1)	(2)	(3)
PROC_{it}	0.055^a	0.073^a	-0.061
	(0.004)	(0.028)	(0.049)
$\log(\text{Credit}_{it-1})$	-0.410^a	-0.175^a	-0.229^a
	(0.001)	(0.043)	(0.044)
Observations	700,780	8,310	3,683
R-squared	0.786	0.360	0.458
Sector×quarter FE	Yes	No	No
Firm×year FE	Yes	Yes	Yes
Quarter FE	No	Yes	Yes
Auction FE	No	Yes	Yes

Notes: Results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (C.1): with firms obtaining at least one procurement project over 2000-13 in column (1), and with firms who participated in procurement contests over 2013–15 in columns (2) and (3), where the PROC dummy indicates the winning firm ('First') in column (2) and the runner-up firm ('Second') in column (3). All regressions use quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

C.2 Effects on impact using data on all bidders

We next use the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to (C.1), except that we can identify the association between a firm's ranking in a given auction and its ensuing credit growth. To be more precise, we run two regressions similar to specification (C.1) at the auction level. In the first regression, we include all bidders and the PROC variable indicates which firm wins the auction ('First' place). Table A.II, column (2), shows the results. We find that the winner of a procurement contract has higher credit growth relative to the firms it competes against in a given auction. Note that identification of the coefficient is exploiting the full time series of bidders, so the comparison is based on the within-auction group of firms but also with respect to each firm's annual credit growth given the inclusion of firm×year effects. The coefficient on the winner is 0.073, which indicates that winning the auction is associated to a 7.3 percentage points higher credit growth annually.

While the bidder firms' sample is more restrictive than the full sample, we are reassured that we are picking up an unbiased "procurement effect" for a few reasons. First, the point estimates of PROC in columns (1) and (2) are remarkably similar, even though the sample and variation exploited are slightly different. Second, we are able to control for firm×year effects in both regressions, thus helping dilute firm-specific productivity or demand effects at the annual level. Third, an identification threat would be that productivity or demand shocks at the quarterly level are correlated with the concession of procurement projects such that winning the contract may be a proxy for these shocks. Therefore, our estimate may capture the effect of being ranked above other firms as opposed to the effect of obtaining the procurement contract. In Column (3) of Table A.II we drop the winner of the procurement contest and the PROC dummy now indicates which firm was runner-up ('Second' place). We run this second regression to make sure that winning the contract, as opposed to the relative ranking, is what is really associated with differences in credit growth across auction participants. The estimated coefficient on PROC implies that there is no statistical difference in quarterly credit growth for the firm that placed second relative to other losers of the auction. Fourth, as in any diff-in-diffs type of environment, it could be the case that winner firms are on different credit trajectories than non-winner firms. In principle, this should be captured by the firm×year effects. We provide evidence below (see Figure A.IV) which shows that this is not the case. In particular, we show that the evolution of credit growth for winners and non-winners was similar before the auction and that it diverged afterwards.

We next decompose the increase in credit associated with winning a procurement contract into that coming from collateralized vs. non-collateralized credit, which will help us motivate the type of financial constraint we use in our model. To this end, we use the information on the composition of firms' loans, which indicates whether these loans require collateral or not to be posted by a firm to receive financing from a bank. We therefore run a similar regression as (C.1), constructing the dependent variable at the firm×credit-type×quarter level, and split the estimation between collateralized and non-collateralized credit growth.

Table A.III presents the main results, where c denotes the additional collateral/non-collateral dimension that we exploit in the data. Looking at the main sample, we see that a procurement contract is not significantly correlated with the growth rate of collateralized credit in column (1). However, when turning to column (2) we see a positive and significant association with a firm obtaining a procurement contract and non-collateralized credit growth. The results with the bidders sample, in columns (3) and (4), mimic the findings for the main sample. That is, a firm winning a contract experiences significantly larger growth in non-collateralized loans relative to losing firms, but there is no differential for collateralized loan growth. Regressions for the second vs. the rest samples in columns (5) and (6) do not yield any significant estimates. Overall, these findings point to the growth rate in overall credit associated with obtaining a procurement contract observed in Table A.II being driven by the growth in loans that do not require tangible-assets backing.

Pre-trends for winners vs. the rest. Graphically, the right panel in Figure A.IV shows the average growth of credit without collateral of firms that win a procurement project in

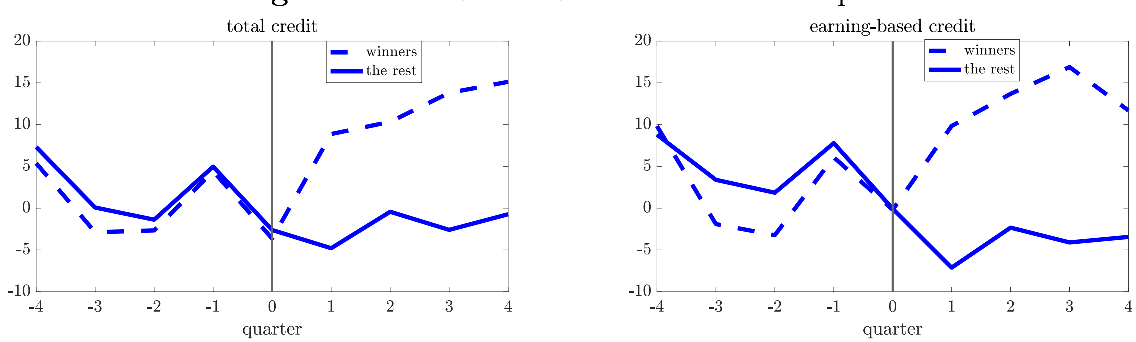
Table A.III. Composition of Credit Growth and Procurement

	All firms		Bidders only			
			First		Second	
	Collat.	NoCollat.	Collat.	NoCollat.	Collat.	NoCollat.
	(1)	(2)	(3)	(4)	(5)	(6)
$PROC_{it}$	0.001	0.070^a	-0.011	0.080^b	-0.019	-0.058
	(0.006)	(0.005)	(0.029)	(0.031)	(0.044)	(0.057)
$\log(Credit_{ict-1})$	-0.474^a	-0.421^a	-0.449^a	-0.192^a	-0.461^a	-0.254^a
	(0.003)	(0.001)	(0.073)	(0.040)	(0.064)	(0.044)
Observations	224,011	557,873	2,690	8,110	1,423	3,606
R-squared	0.791	0.764	0.357	0.368	0.435	0.435
Sector×quarter FE	Yes	Yes	No	No	No	No
Firm×year FE	Yes	Yes	Yes	Yes	Yes	Yes
Quarter FE	No	No	Yes	Yes	Yes	Yes
Auction FE	No	No	Yes	Yes	Yes	Yes

Notes: Results from estimating the relationship between collateralized (Collat.) and non-collateralized (NonCollat.) credit growth and procurement participation (PROC) by regression (C.1) with firms obtaining at least one procurement project over 2000-13 in columns (1) and (2), and with firms who participated in procurement contests over 2013-15 in columns (3)-(6) respectively, where the PROC dummy indicates the winning firm ('First') in columns (3)-(4) and the runner-up firm ('Second') in columns (5)-(6). All regressions use quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Figure A.IV. Credit Growth: bidders sample



Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

C.3 Loan applications

In this Section we ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España's credit register allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \tag{C.2}$$

where the variable 'Loan granted' is a dummy variable equal to 1 when firm i receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect α_{bt}, and for macroeconomic events with sector×quarter fixed effects α_{st}.

Table A.IV. Probability of a New Loan and Procurement

	All firms	
	(1)	(2)
PROC_{it}	0.024^{a}	0.023^{b}
	(0.008)	(0.011)
Observations	36,857	26,924
R-squares	0.395	0.628
Firm×bank FE	Yes	Yes
Bank×quarter FE	No	Yes
Sector×quarter FE	No	Yes

Notes: Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (C.2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered at the firm level; ^{a} indicates significance at the 1% level, ^{b} at the 5% level, and ^{c} at the 10% level.

Table A.IV shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

D Steady state equilibrium

Let $\mathbf{X} \equiv S \times A \times \{0,1\}$ be the state space of the household problem, $\mathbf{X}_1 \equiv S \times A \times \{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Given government policy parameters Y_g and m_g and a distribution of entrants Γ_0, we define the steady state equilibrium of the model as (i) firms' policy functions, (ii) the probability measure Γ, (iii) total amount of final private good Y_p, (iv) interest rate r, (v) tax rate τ, (vi) relative price of the final public good P_g, and (vii) the procurement probability shifter η_0, so that:

a) Entrepreneurs solve their optimization problem

b) The probability measure Γ is stationary

c) The market for the private good clears:

$$\int_{\mathbf{X}} p_p(s,a,d)\, u(s,a,d)\, y(s,a,d)\, d\Gamma = Y_p = \int_{\mathbf{X}} \left[b(s,a,d) + c(s,a,d) + \delta k(s,a,d)\right] d\Gamma$$

d) The market for the public good clears: $\int_{\mathbf{X}_1} p_g(s,a,1)\left[1-u(s,a,1)\right] y(s,a,1)\, d\Gamma = P_g Y_g$

e) The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector: $\int_{\mathbf{X}} Pr\left(d'=1 \mid b(s,a,d)\right) d\Gamma = \int_{\mathbf{X}_1} d\Gamma = m_g$

f) The budget constraint of the government holds:

$$P_g Y_g = rD + \tau \int_{\mathbf{X}} \pi(s,a,d)\, d\Gamma + (1-\theta)\left[\int_{\mathbf{X}} a'(s,a,d) d\Gamma - \int_{\mathbf{X}} a d\Gamma_0\right]$$

g) By Walras law, the credit market clears: $D = \int_{\mathbf{X}} \left[k(s,a,d) - a(s,a,d)\right] d\Gamma$

Several comments are in order. First, the parameter η_0 driving the average probability of a procurement project is an equilibrium object that ensures meeting equilibrium condition (e). It captures in reduced form the competition for projects. Second, the government can accumulate financial wealth D, which serves as an aggregate counterpart for the loans of entrepreneurs such that loans do not need to be in zero net supply in condition (g). Indeed, we calibrate D to match an interest rate of $r = 5\%$ given the total amount of debt relative to capital held by firms in the data. Third, condition (f) establishes that the government budget constraint in steady state is such that procurement is financed by taxes, plus interest revenues from the stationary amount of government wealth D, plus accidental bequests left by dying entrepreneurs, minus the initial net worth provided by the government to newly born entrepreneurs (which is dictated by the exogenously fixed distribution of entrants Γ_0). Finally, we note that the aggregate objects determined in general equilibrium that are relevant for the optimization problem of households are Y_p, r, τ, P_g, and η_0.

E Details on the static production problem

In this Appendix we characterize the solution of the static production problem. First, in Section E.1 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, in Section E.2 we characterize analytically the solution to the production problem for firms without procurement ($d = 0$), which is useful to understand the interaction of asset based and earnings based financial constraints. Next, in Section E.3 we characterize analytically some of the solutions to the production problem for firms with procurement ($d = 1$) for the case $\sigma_p = \sigma_g$. Finally, in Section E.4 we show analytically the effect of a procurement shock for the case $\sigma_p = \sigma_g$, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

Before going to all these results, we start the Appendix by rewriting the FOC of the static production problem as follows. First, note that because the FOC for u states that the marginal revenue per unit of output sold —including its value as collateral— has to be equalized across the two sectors, and using the fact that $\frac{\partial p_p y_p}{\partial k} / \frac{\partial p_p y_p}{\partial u} = u/k$ and $\frac{\partial p_g y_g}{\partial k} / \frac{\partial p_g y_g}{\partial (1-u)} = (1-u)/k$ we can write the FOC for k as:

$$\text{MRPK}_p \equiv \frac{\partial p_p y_p}{\partial k_p} = \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \tag{E.1}$$

or as

$$\text{MRPK}_g \equiv \frac{\partial p_g y_g}{\partial k_g} = \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \tag{E.2}$$

or combining them both, $\text{MRPK} \equiv \frac{\partial [p_p y_p + p_g y_g]}{\partial k} = u \left(\frac{r+\delta+\lambda}{1+\lambda\phi_p} \right) + (1-u) \left(\frac{r+\delta+\lambda}{1+\lambda\phi_g} \right)$, where note that $\frac{\partial p_p y_p}{\partial k_p} = \frac{\partial p_p y_p}{\partial k} \frac{1}{u}$ and $\frac{\partial p_g y_g}{\partial k_g} = \frac{\partial p_g y_g}{\partial k} \frac{1}{1-u}$. That is, the revenue marginal product of capital in each sector (MRPK_p and MRPK_g) is equal to the capital cost of each sector and the revenue marginal product of capital for the whole firm (MRPK) is a weighted average of the capital costs in the two sectors, with the weights given but the cost shares of each sector.

It will be useful later on to use the actual revenue functions and substitute in equations (E.1) and (E.2) to obtain,

$$\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{p_p y_p}{k} \; \frac{1}{u} \; = \frac{r + \delta + \lambda}{1 + \phi_p \lambda} \tag{E.3}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g} \right) \frac{p_g y_g}{k} \frac{1}{1-u} = \frac{r + \delta + \lambda}{1 + \phi_g \lambda} \tag{E.4}$$

and using the production function one can write them as

$$\left(\frac{\sigma_p - 1}{\sigma_p} \right) p_p \, s = \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \tag{E.5}$$

$$\left(\frac{\sigma_g - 1}{\sigma_g} \right) p_g \, s = \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \tag{E.6}$$

Finally, dividing these two equations we get an expression for the optimal relative prices,

$$\frac{p_p}{p_g} = \frac{1 + \lambda\phi_g}{1 + \lambda\phi_p}\frac{(\sigma_g - 1)/\sigma_g}{(\sigma_p - 1)/\sigma_p} \tag{E.7}$$

Note that whenever $\sigma_p = \sigma_g$, $p_g/p_p = 1$ for firms without binding financial frictions ($\lambda = 0$). For firms with binding financial frictions ($\lambda > 0$) $p_g/p_p < 1$ ($p_g/p_p > 1$) whenever $\phi_g > \phi_p$ ($\phi_g < \phi_p$) because production is shifted towards the sector that provides better collateral, and $p_g/p_p = 1$ whenever $\phi_g = \phi_p$.

E.1 Some preliminary results

Lemma 1 *The terms $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below $1/\phi_p$ and $1/\phi_g$ respectively, (b) increasing in λ, and (c) strictly above $r + \delta$ when $\lambda > 0$, if and only if $\phi_p < (\delta + r)^{-1}$ and $\phi_g < (\delta + r)^{-1}$ respectively.*

Proof: Part (a) is straightforward:

$$\frac{r + \delta + \lambda}{1 + \lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p (r + \delta + \lambda) < (1 + \lambda\phi_p) \Leftrightarrow \phi_p (r + \delta) < 1 \Leftrightarrow \phi_p < (r + \delta)^{-1}$$

For part (b) note that

$$\frac{d}{d\lambda}\left(\frac{r + \delta + \lambda}{1 + \lambda\phi_p}\right) \propto (1 + \lambda\phi_p) - \phi_p (r + \delta + \lambda) > 0 \Leftrightarrow \phi_p (r + \delta) < 1 \Leftrightarrow \phi_p < (r + \delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r + \delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ∎

Proposition 1 *Holding s constant, more constrained firms sell less to the private sector, sell less to the public sector, and demand less capital if both $\phi_p < (\delta + r)^{-1}$ and $\phi_g < (\delta + r)^{-1}$.*

Proof: Let's combine the FOC (E.3) with the demand equation to produce the expression,

$$y_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p s \frac{1 + \lambda\phi_p}{r + \delta + \lambda}\right)^{\sigma_p}$$

Then, by virtue of Lemma 1 y_p falls with λ whenever $\phi_p < (\delta + r)^{-1}$. The case for y_g is analogous. Finally, note that total output is split between private sector and public sector sales, that is, $y_p + y_g = f(s, k) = sk$, so the derivative of capital with respect to λ is just,

$$\frac{dk}{d\lambda} = \frac{1}{s}\left(\frac{dy_p}{d\lambda} + \frac{dy_g}{d\lambda}\right)$$

which is negative given the previous results in this Proposition. ∎

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if and only if* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$ *respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (E.3) when $\lambda = 0$, which implies $\frac{p_p y_p}{k}\frac{1}{u} = \frac{\sigma_p}{\sigma_p - 1}(r+\delta)$. When $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < \phi_p^{-1}$ this leads to $\frac{p_p y_p}{k}\frac{1}{u} < \phi_p^{-1} \Leftrightarrow \phi_p p_p y_p < uk$, that is, the optimal unconstrained capital for the private sector, uk, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (E.4) ■

Proposition 2 *Entrepreneurs with zero net worth are financially constrained if both* $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$.

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p - 1}(r+\delta) < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g - 1}(r+\delta) < 1$, then following Lemma 2 both $\phi_p p_p y_p < uk$ and $\phi_g p_g y_g < (1-u)k$. Adding them up leads to $\phi_p p_p y_p + \phi_g p_g y_g < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The term* $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}$ *describing the share of capital that can be self-financed through revenues is positive and strictly smaller than one for constrained firms.*

Proof: That this term is positive is straightforward. To show that it is lower than one, note that for constrained firms the borrowing constraint holds with equality. Hence, for $a \geq 0$ it must be that $k \geq \phi_p p_p y_p + \phi_g p_g y_g$ or $\phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ (with strict equality for $a = 0$). Given our revenue function, the marginal products are proportional to the average products $\frac{\partial p_p y_p}{\partial k} = \left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{y_p p_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{y_g p_g}{k}$, so we can rewrite

$$\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} = \phi_p \left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{p_p y_p}{k} + \phi_g \left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{p_g y_g}{k}$$

Note that $\sigma_p > 1$ and $\sigma_g > 1$ implies $\frac{\sigma_p - 1}{\sigma_p} < 1$ and $\frac{\sigma_g - 1}{\sigma_g} < 1$ (the marginal products are below the average products), and hence it is the case that $\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k} < \phi_p \frac{p_p y_p}{k} + \phi_g \frac{p_g y_g}{k} \leq 1$ ■

Lemma 4 *The term* $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ *describing the increase in credit that can be achieved by reallocation output to the private sector has the sign of* $(\phi_p - \phi_g)$ *for constrained firms.*

Proof: Using the FOC for u, we can write:

$$\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u} = \frac{\partial p_p y_p}{\partial u}\left[\phi_p - \phi_g \frac{1 + \lambda \phi_p}{1 + \lambda \phi_g}\right] = \frac{\partial p_p y_p}{\partial u}\phi_g \left[\frac{\phi_p}{\phi_g} - \frac{\lambda^{-1} + \phi_p}{\lambda^{-1} + \phi_g}\right]$$

Note that with $\phi_p > \phi_g$ ($\phi_p < \phi_g$), this expression is positive (negative) when λ tends to zero, it decreases (increases) monotonically with λ, and tends to zero when λ tends to infinity. ■

E.2 Firms without procurement

We start analyizing the production problem for firms without procurement ($d = 0$).

E.2.1 Unconstrained firms

With $\lambda = 0$ the FOC for k becomes, $\frac{\partial p_p y_p}{\partial k} = r + \delta$, i.e., firms must equalize the marginal revenue product of capital to the cost of capital. This equation defines the optimal demand of capital $k^* (s, a, 0)$ for every entrepreneur of type $(s, a, d = 0)$. In particular, one gets $\frac{\sigma - 1}{\sigma} \frac{p_p y_p}{k} = r + \delta$ and substituting for the revenue function yields the optimal demand for capital

$$k^* (s, a, 0) = \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r + \delta} \right]^{\sigma} s^{\sigma - 1} \tag{E.8}$$

Next, note that profits are given by $\pi = p_p y_p - (r + \delta) k$, which given the optimal choice of capital can be written as $\pi = \frac{1}{\sigma_p} p_p y_p$ or $\pi = \frac{1}{\sigma_p - 1} (r + \delta) k$. Substituting optimal capital demand to the revenue function gives $p_p y_p = B_p \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{B_p}{r + \delta} \right]^{\sigma_p - 1} s^{\sigma_p - 1}$, which can be substituted back to the profit function to obtain:

$$\pi^* (s, a, 0) = \frac{1}{\sigma_p} \left[\left(\frac{\sigma_p - 1}{\sigma_p} \right) \frac{1}{r + \delta} \right]^{\sigma_p - 1} B_p^{\sigma} s^{\sigma_p - 1} \tag{E.9}$$

Hence, capital demand and profits increase monotonically with the shock s and are independent from net worth a.

E.2.2 Constrained firms

If the firm is constrained, then $\lambda > 0$ and the FOC of the problem are:

$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k} = r + \delta + \lambda \tag{E.10}$$

$$k = \phi_a a + \phi_p p_p y_p \tag{E.11}$$

which determine k and λ. In particular, the borrowing constraint (E.11) defines capital demand $k (s, a, 0)$, the FOC (E.10), delivers the shadow value of the constraint $\lambda (s, a, 0)$, and the objective function delivers the profit function $\pi (s, a, 0)$. The next propositions characterize the derivatives of these functions with respect to the state variables a and s.

Totally differentiating equation (E.11) in turn with respect to a and s yields,

$$\frac{\partial k (s, a, 0)}{\partial a} = \phi_a \left(1 - \phi_p \frac{\partial p_p y_p}{\partial k} \right)^{-1} \tag{E.12}$$

$$\frac{\partial k (s, a, 0)}{\partial s} = \phi_p \frac{\partial p_p y_p}{\partial s} \left(1 - \phi_p \frac{\partial p_p y_p}{\partial k} \right)^{-1} \tag{E.13}$$

With $\phi_p = 0$ we are in the case without earnings-based collateral constraints and these derivatives are just equal to ϕ_a and 0 respectively: higher net worth allows to operate with more capital but higher productivity does not. With $\phi_p > 0$ both derivatives are positive, that is, constrained firms with more net worth or higher productivity operate with more capital. Indeed, in this case $\frac{\partial k(s,a,0)}{\partial a} > \phi_a$ because an increase in net worth has a multiplier effect through the increase in revenues and the easing of the earnings-based financial constraint (see Lemma 3). This is stated in the next proposition:

Proposition 3 *The derivative of $k(s,a,0)$ w.r.t. a is positive, while the derivative of $k(s,a,0)$ w.r.t. s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: The derivatives of $k(s,a,0)$ with respect to a and s are given by equation (E.12) and (E.13). $\phi_a \geq 1$ and Lemma 3 states that $\phi_p \frac{\partial p_p y_p}{\partial k} < 1$, so the derivative with respect to a is strictly positive. For the derivative with respect to s, note additionally that $\frac{\partial p_p y_p}{\partial s} > 0$. Hence, this derivative is strictly positive (zero) if $\phi_p > 0$ ($\phi_p = 0$). ■

Note also that the derivatives of capital with respect to a and s are higher for more constrained firms (higher λ) because the multiplier effect of the earnings-based constraints is larger for firms with higher marginal product of capital, that is, the increase in capital demand with net worth a or productivity s is larger for more financially constrained firms.

Corollary 1 *The derivatives of $k(s,a,0)$ w.r.t. a and s increase with λ.*

Proof: The derivatives are characterized by equations (E.12) and (E.13). Using the FOC (E.10) and the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can further rewrite them as

$$\frac{\partial k(s,a,0)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.14}$$

$$\frac{\partial k(s,a,0)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}\left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.15}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (E.31) and (E.32) increases with λ, which is proved in Lemma 1. ■

Next, (E.10) allows to recover $\lambda(s,a,0)$. It can be shown that $\lambda(s,a,0)$ declines with a —wealthier entrepreneurs can finance larger amounts of capital and are hence less constrained— and increases with s —s increases optimal capital by more than it increases the amount of capital that can be self-financed through revenues. This is stated formally in Proposition 4.

Proposition 4 *The derivative of $\lambda(s,a,0)$ w.r.t. a is negative, while w.r.t. s it is positive as long as $a > 0$ (and zero otherwise).*

Proof: Equation (E.10) can be rewritten as $\frac{\partial p_p y_p}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$. The r.h.s, the cost of capital, increases with λ, see Lemma 1. Hence, the sign of the derivative of $\lambda(s,a,0)$ with respect to a or s equals the sign of the derivative of $\frac{\partial p_p y_p}{\partial k}$ with respect to a or s. We start by obtaining an expression of the marginal revenue product of capital by use of the revenue function:

$$\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k} = \frac{\sigma_p - 1}{\sigma_p} B_p s^{\frac{\sigma_p - 1}{\sigma_p}} k^{-\frac{1}{\sigma_p}}$$

where $\frac{\partial p_p y_p}{\partial k}$ declines with $k(s,a,0)$. For net worth a it is straightforward to see that $\lambda(s,a,0)$ declines with a because $k(s,a,0)$ increases with a, see Proposition 3. For the shock s we take the derivative of the marginal revenue product of capital w.r.t. s, and asking it to be non-negative delivers:

$$\frac{\partial^2 p_p y_p}{\partial k \partial s} \propto \left[(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k}\right] \geq 0$$

where the first term reflects the positive direct effect of s on the marginal revenue product of capital for fixed capital, while the second term reflects the negative indirect effect of s on the marginal revenue product of capital through its induced increase in the choice of capital. Using $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$, equation (E.13) shows that

$$\frac{\partial k}{\partial s}\frac{s}{k} = \phi_p \frac{\partial p_p y_p}{\partial k}\left(1 - \phi_p \frac{\partial p_p y_p}{\partial k}\right)^{-1}$$

Then, we can rewrite

$$(\sigma_p - 1) - \frac{\partial k}{\partial s}\frac{s}{k} \geq 0 \quad \Leftrightarrow \quad \phi_p \frac{\partial p_p y_p}{\partial k} \leq \frac{\sigma_p - 1}{\sigma_p} \quad \Leftrightarrow \quad k \geq \phi_p p_p y_p$$

where the last step uses the fact that $\frac{\partial p_p y_p}{\partial k} = \frac{\sigma_p - 1}{\sigma_p}\frac{p_p y_p}{k}$. Note that whenever a firm has zero net worth it will be able to self-finance capital up to the point $k = \phi_p p_p y_p$. In this case the derivative of $\lambda(s,a,0)$ with respect to s will be zero. Whenever a firm owns $a > 0$ then capital k is going to be above $\phi_p p_p y_p$ and the derivative of $\lambda(s,a,0)$ w.r.t. s is positive. ■

Next, with Corollary 1 and Proposition 4, one can also show that $\frac{\partial^2 k(s,a,0)}{\partial a^2} < 0$ (the increase in capital due to an increase in net worth is larger for firms with less net worth) and that $\frac{\partial^2 k(s,a,0)}{\partial a \partial s} > 0$ (the increase in capital due to an increase in net worth is larger for firms with higher productivity), see Corollary 2.

Corollary 2 *The derivative of $\partial k(s,a,0)/\partial a$ w.r.t. a is negative, while the derivative of $\partial k(s,a,0)/\partial a$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,0)}{\partial a^2} = \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial a}$$
$$\frac{\partial^2 k(s,a,0)}{\partial a \partial s} = \frac{\partial^2 k(s,a,0)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,0)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 1. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,0)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,0)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,0)}{\partial a}$ and $\frac{\partial \lambda(s,a,0)}{\partial s}$ described in Proposition 4. ■

Finally, we can also characterize the derivatives of the profit function $\pi(s,a,0)$, as

$$\frac{\partial \pi(s,a,0)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right] \frac{\partial k(s,a,0)}{\partial a} \tag{E.16}$$

$$\frac{\partial \pi(s,a,0)}{\partial s} = \left[\frac{\partial p_p y_p}{\partial k} - (r+\delta)\right] \frac{\partial k(s,a,0)}{\partial s} + \frac{\partial p_p y_p}{\partial s} \tag{E.17}$$

We can substitute the partial derivatives of capital w.r.t. a and s described by (E.12) and (E.13) into equations (E.16) and (E.17) respectively. Then, using the FOC in (E.11) yields

$$\frac{\partial \pi(s,a,0)}{\partial a} = \phi_a \lambda(s,a,0) \tag{E.18}$$

$$\frac{\partial \pi(s,a,0)}{\partial s} = (1 + \phi_p \lambda(s,a,0)) \frac{\partial p_p y_p}{\partial s} \tag{E.19}$$

Profits increase with a since more net worth allows to increase constrained capital. They increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$, a larger s implies higher revenues and hence more borrowing and higher capital. For this, see the next Proposition, with a Corollary on second derivatives.

Proposition 5 *The derivatives of $\pi(s,a,0)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.18) and (E.19). These derivatives are positive because $\lambda(s,a,0) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ (see the revenue function). ■

Corollary 3 *The derivative of $\partial \pi(s,a,0)/\partial a$ w.r.t. a is negative, while the derivative of $\partial \pi(s,a,0)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using equation (E.18) we can write the second derivatives as $\frac{\partial^2 \pi(s,a,0)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial a}$ and $\frac{\partial^2 \pi(s,a,0)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,0)}{\partial s}$. Then, one only needs to check the signs of the derivatives of λ in Proposition 4. ■

E.2.3 Binding constraints

Finally, we need to characterize the set of entrepreneurs that are financially constrained. Under Assumption 1, Proposition 2 says that $k(s,0,0) < k^*(s,0,0)$, and we have shown that $\frac{\partial k(s,a,0)}{\partial a} > 0$ and that $k^*(s,a,0)$ is invariant in a. Hence, for every s there will be a unique threshold $\underline{a}(s,0)$ satisfying $k(s,a,0) = k^*(s,a,0)$ such that for every s entrepreneurs with $a \geq \underline{a}(s,0)$ are unconstrained while entrepreneurs with $a < \underline{a}(s,0)$ are constrained.

E.3 Firms with procurement

We now analyze the production problem for firms with procurement ($d = 1$), given $\sigma_p = \sigma_g = \sigma$.

E.3.1 Unconstrained firms

With $\lambda = 0$ the FOC for k and u become $\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u} = 0$ and $\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = r + \delta$, which states that unconstrained firms allocate output between the two sectors to equalize the marginal revenues and choose capital such that the marginal revenue product of capital equals the capital costs. These two equations determine the optimal capital demand $k^* (s, a, 1)$ and allocation of output in the private sector $u^* (s, a, 1)$ for entrepreneurs of type $(s, a, d = 1)$. In particular, the FOC for k can be written as $\frac{\sigma-1}{\sigma} \frac{p_p y_p + p_g y_g}{k} = r + \delta$. Substituting for the revenue functions yields the optimal demand for capital $k^* (s, a, 1) = \left[\left(\frac{\sigma-1}{\sigma}\right) \frac{1}{r+\delta}\right]^{\sigma} \left(B_p^{\sigma} + B_g^{\sigma}\right) s^{\sigma-1}$. Using the FOC for u implies $\frac{p_p y_p}{ku} = \frac{p_g y_g}{k(1-u)}$ where substituting the revenue functions yields:

$$u^* (s, a, 1) \quad = \quad \left(1 + \left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \tag{E.20}$$

Clearly $k^* (s, a, 1)$ increases monotonically with the shock s and is invariant with the net worth a, while $u^* (s, a, 1)$ is independent from both s and a and is only determined by the relative demands B_p / B_g. Next, note that profits are given by $\pi = p_p y_p + p_g y_g - (r + \delta) k$, which given the condition for the optimal choice of capital can be written as $\pi = \frac{1}{\sigma} (p_p y_p + p_g y_g)$ or $\pi = \frac{1}{\sigma-1} (r + \delta) k$. Substituting the optimal capital demand into the revenue function gives total revenues as $p_p y_p + p_g y_g = \left[\left(\frac{\sigma-1}{\sigma}\right) \frac{1}{r+\delta}\right]^{\sigma-1} \left(B_p^{\sigma} + B_g^{\sigma}\right) s^{\sigma-1}$, which can be substituted back into the profit function to obtain

$$\pi^* (s, a, 1) \quad = \quad \frac{1}{\sigma} \left[\left(\frac{\sigma - 1}{\sigma}\right) \frac{1}{r + \delta}\right]^{\sigma-1} \left(B_p^{\sigma} + B_g^{\sigma}\right) s^{\sigma-1} \tag{E.21}$$

The profit function increases with productivity s and is invariant with assets a.

E.3.2 Constrained firms.

We now explain how to determine $k (s, a, 1)$, $u (s, a, 1)$, and $\lambda (s, a, 1)$ for constrained firms. The characterization of these functions is simple whenever $\phi_g = \phi_p$ and more involved when not. To characterize $u (s, a, 1)$ let's start by noting that the FOC for u can be rewritten as in (E.7) and that after substituting prices we obtain,

$$\frac{u}{1 - u} = \left(\frac{B_p}{B_g}\right)^{\sigma} \left(\frac{1 + \lambda \phi_p}{1 + \lambda \phi_g}\right)^{\sigma} \tag{E.22}$$

To characterize $k(s,a,1)$ we totally differentiate the binding borrowing constraint with respect to a and s in turn, which gives,

$$\frac{\partial k}{\partial a} = \left[\phi_a + \left(\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}\right)\frac{du}{da}\right]\left[1 - \left(\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.23}$$

$$\frac{\partial k}{\partial s} = \left[\left(\phi_p \frac{\partial p_p y_p}{\partial s} + \phi_g \frac{\partial p_g y_g}{\partial s}\right) + \left(\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}\right)\frac{du}{ds}\right]\left[1 - \left(\phi_p \frac{\partial p_p y_p}{\partial k} + \phi_g \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \tag{E.24}$$

Finally, the derivatives of the profit function $\pi(s,a,1)$ are given by

$$\frac{\partial \pi(s,a,1)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial a} + \left[\frac{\partial p_p y_p}{\partial u} + \frac{\partial p_g y_g}{\partial u}\right]\frac{\partial u(s,a,1)}{\partial a} \tag{E.25}$$

$$\begin{aligned}\frac{\partial \pi(s,a,1)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} - (r+\delta)\right]\frac{\partial k(s,a,1)}{\partial s} \\ &+ \left[\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right]\frac{\partial u(s,a,1)}{\partial s} + \frac{\partial p_p y_p}{\partial s}\end{aligned} \tag{E.26}$$

Now, substituting (E.1), (E.2), and (E.23) into (E.25) and using the FOC for u we obtain

$$\frac{\partial \pi(s,a,1)}{\partial a} = \phi_a \lambda(s,a,1) \tag{E.27}$$

while substituting (E.1), (E.2), and (E.24) into (E.26) and using the FOC for u we obtain

$$\frac{\partial \pi(s,a,1)}{\partial s} = (1 + \phi_p \lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1 + \phi_g \lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} \tag{E.28}$$

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits.

For the case $\phi_g = \phi_p$ it can be shown that $u(s,a,1) = u^*(s,a,1)$ —as revenues from both sectors are equally pledgeable— and hence $u(s,a,1)$ is invariant in a and s. This makes the problem analogous to the case without procurement ($d = 0$), and hence the derivatives of $k(s,a,1)$, $\lambda(s,a,1)$, and $\pi(s,a,1)$ with respect to a and s are as in the $d = 0$ case. This can be seen in the next propositions.

Proposition 6 *When $\phi_g = \phi_p$, the optimal choice of $u(s,a,1)$ is as in the unconstrained case and it is hence independent from a and s.*

Proof: Equation (E.22) clearly shows that whenever $\phi_g = \phi_p$ the optimal solution for u for constrained firms is equal to the one for unconstrained firms, see equation (E.20). This means that $u(s,a,1)$ is independent from s and a and only determined by the relative demands B_p/B_g of each sector. ■

Proposition 7 *When $\phi_g = \phi_p$, the derivative of $k(s,a,1)$ w.r.t. a is positive, while the derivative of $k(s,a,1)$ w.r.t. s is positive as long as $\phi_p > 0$ (and zero othwewise).*

Proof: Note that with $\phi_g = \phi_p$ the optimality condition for u implies that $\frac{\partial p_g y_g}{\partial u} = -\frac{\partial p_p y_p}{\partial u}$ and hence we can rewrite equations (E.23) and (E.24) as follows,

$$\frac{\partial k}{\partial a} = \phi_a \left[1 - \phi_p \left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \quad \text{(E.29)}$$

$$\frac{\partial k}{\partial s} = \phi_p \left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right)\left[1 - \phi_p \left(\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k}\right)\right]^{-1} \quad \text{(E.30)}$$

Given $\phi_a \geq 1$ and $\phi_p > 0$ both $\partial k/\partial a$ and $\partial k/\partial s$ are positive by Lemma 3. If $\phi_p = 0$ then $\partial k/\partial s = 0$. ■

Corollary 4 *When $\phi_g = \phi_p$, the derivatives of $k(s,a,1)$ w.r.t. a and s increase with λ.*

Proof: The FOC for k can be written as $\frac{\partial p_p y_p}{\partial k} + \frac{\partial p_g y_g}{\partial k} = \frac{r+\delta+\lambda}{1+\lambda\phi_p}$. Then, using the fact that $\frac{\partial p_p y_p}{\partial s} = \frac{k}{s}\frac{\partial p_p y_p}{\partial k}$ we can rewrite equations (E.29) and (E.30) as

$$\frac{\partial k(s,a,1)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \quad \text{(E.31)}$$

$$\frac{\partial k(s,a,1)}{\partial s} = \phi_p \frac{k}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}\left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \quad \text{(E.32)}$$

To prove this corollary it is enough to show that the term $(r+\delta+\lambda)/(1+\lambda\phi_p)$ in equations (E.31) and (E.32) increases with λ, which is proved in Lemma 1. ■

Proposition 8 *When $\phi_g = \phi_p$, the derivative of $\lambda(s,a,1)$ w.r.t. a is negative, while the derivative of $\lambda(s,a,1)$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Note that the FOC for k_p is given by equation (E.1). Because u is invariant in a and s, see Proposition 6, the proof of Proposition 8 for the case $d = 0$ carries over. ■

Corollary 5 *When $\phi_g = \phi_p$, the derivative of $\partial k(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial k(s,a,1)/\partial a$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: By the chain rule we can write

$$\frac{\partial^2 k(s,a,1)}{\partial a^2} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial a}$$

$$\frac{\partial^2 k(s,a,1)}{\partial a \partial s} = \frac{\partial^2 k(s,a,1)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,1)}{\partial s}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 4. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,1)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,1)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial \lambda(s,a,1)}{\partial s}$ described in Proposition 8. ■

Proposition 9 *When $\phi_g = \phi_p$, the derivatives of $\pi(s,a,1)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.27) and (E.28). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 6 *When $\phi_g = \phi_p$, the derivative of $\partial\pi(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial\pi(s,a,1)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using (E.27) we can write the second derivatives as, $\frac{\partial^2\pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial a}$ and $\frac{\partial^2\pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial\lambda(s,a,1)}{\partial s}$. The signs of the derivatives of λ follow from Proposition 8. ■

The case $\phi_g > \phi_p$ is more involved because $u(s,a,1)$ changes with a and s. It can be shown that firms with more net worth are less constrained and hence run larger firms and sell a higher fraction of output to the private sector, which offers lower collateral value. More productive firms are able to run larger firms thanks to the earnings-based constraints but are more constrained —because their optimal capital is even larger— and hence sell a lower fraction of output to the private sector. This means that firms with larger s sell a larger quantity to the public sector but they may either sell a larger or smaller quantity to the private sector. This is proved in the following propositions.

Lemma 5 *The sign of the derivative of u w.r.t. λ is the same as the sign of $(\phi_p - \phi_g)$, that is, more constrained firms shift their output relatively towards the sector whose revenues provide better collateral.*

Proof: Equation (E.22) implies $du/d\lambda < 0$ when $\phi_g > \phi_p$ and the opposite when $\phi_g < \phi_p$. ■

Proposition 10 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ w.r.t. a are positive, positive, and negative respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, using the FOC (E.3) and (E.4), the demand equations, and the production function we can write,

$$k_p = \left(\frac{\sigma_p - 1}{\sigma_p} B_p \frac{1 + \lambda\phi_p}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1} \quad \text{and} \quad k_g = \left(\frac{\sigma_p - 1}{\sigma_p} B_g \frac{1 + \lambda\phi_g}{r + \delta + \lambda}\right)^{\sigma_p} s^{\sigma_p - 1}$$

Adding them up, by the chain rule, let us express: $\frac{\partial k}{\partial a} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial a}$. Also, using equation (E.22) and the chain rule we can write $\frac{\partial u}{\partial a} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial a}$. These two expressions state that $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ should have the same sign because both k and u fall with λ. Given this, equation (E.23) implies that $\frac{\partial k}{\partial a} > 0$ and $\frac{\partial u}{\partial a} > 0$. To see why, recall that by Lemma 3 the denominator is positive. In addition, the term $\phi_p \frac{\partial p_p y_p}{\partial u} + \phi_g \frac{\partial p_g y_g}{\partial u}$ is negative whenever $\phi_g > \phi_p$ see Lemma 4. Hence, for $\frac{\partial k}{\partial a} < 0$ we would need $\frac{\partial k}{\partial u} > 0$. That is, given that higher a allows to increase capital through ϕ_a, for higher a to lead to lower capital it must be that entrepreneurs with higher a tilt production towards the sector with lower collateral value. But this would require the signs of $\frac{\partial k}{\partial a}$ and $\frac{\partial u}{\partial a}$ to be different. Instead, $\frac{\partial k}{\partial a} > 0$ can be obtained with $\frac{\partial u}{\partial a} > 0$. It follows that, because $\frac{\partial k}{\partial \lambda} < 0$ and $\frac{\partial k}{\partial a} > 0$, it must be the case that $\frac{\partial \lambda}{\partial a} < 0$. ■

Proposition 11 *When $\phi_g > \phi_p$, the derivatives of $u(s,a,1)$, $k(s,a,1)$, and $\lambda(s,a,1)$ w.r.t. s are negative, positive, and positive respectively,*

Proof: First note that, following Lemma 5, $du/d\lambda < 0$ when $\phi_g > \phi_p$ and that Proposition 1 says that $dk/d\lambda < 0$. That is, more constrained entrepreneurs tilt production towards the sector with higher collateral value and run smaller firms. Next, by the chain rule (see proof of Proposition 10) we can write $\frac{dk}{ds} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial s} + \frac{\partial k}{\partial s}$ and $\frac{du}{ds} = \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial s}$. We learn two things from here. First, $\frac{dk}{ds} \leq 0$ requires $\frac{\partial \lambda}{\partial s} > 0$ (because $\frac{\partial k}{\partial s} > 0$ and $\frac{\partial k}{\partial \lambda} < 0$). Second, $\frac{\partial \lambda}{\partial s} > 0$ requires $\frac{du}{ds} < 0$ (because $du/d\lambda < 0$). But equation (E.24) shows that if $\frac{du}{ds} < 0$ then it must be $\frac{dk}{ds} > 0$ so this enters a contradiction. Therefore, $\frac{dk}{ds} > 0$. Note that from equation (E.24) $\frac{dk}{ds} > 0$ can be achieved with any sign of $\frac{du}{ds}$. Now, regarding the derivatives of $u(s,a,1)$ and $\lambda(s,a,1)$ with respect to s, two different things can happen. If $\frac{\partial \lambda}{\partial s} \geq 0$ then $\frac{du}{ds} \leq 0$ (this is an if and only if statement), and then $\frac{dk}{ds} > 0$ according to equation (E.24). Instead, if $\frac{\partial \lambda}{\partial s} < 0$ then $\frac{du}{ds} > 0$ (again an if and only if statement) and we can have both $\frac{dk}{ds} > 0$ or $\frac{dk}{ds} < 0$ according to equation (E.24). ■

Proposition 12 *When $\phi_g > \phi_p$, the derivatives of $\pi(s,a,1)$ w.r.t. a and s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.27) and (E.28). These derivatives are positive because $\lambda(s,a,1) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Corollary 7 *When $\phi_g > \phi_p$, the derivative of $\partial\pi(s,a,1)/\partial a$ w.r.t. a is negative, while the derivative of $\partial\pi(s,a,1)/\partial s$ w.r.t. s is positive as long as $a > 0$ (and zero otherwise).*

Proof: Using (E.27) we can write the second derivatives as, $\frac{\partial^2 \pi(s,a,1)}{\partial a^2} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial a}$ and $\frac{\partial^2 \pi(s,a,1)}{\partial a \partial s} = \phi_a \frac{\partial \lambda(s,a,1)}{\partial s}$ The signs of the derivatives of λ follow from Propositions 10 and 11. ■

E.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$

state with firms in the $(s,a,0)$ state.

E.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by $\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^\sigma = \frac{1}{u^*(s,a,1)}$ which implies that $u^*(s,a,1)\, k^*(s,a,1) = k^*(s,a,0)$. Hence, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*(s,a,1) - k^*(s,a,0)$ is given by $\left(\frac{B_g}{B_p}\right)^\sigma k^*(s,a,0)$. Because $k^*(s,a,0)$ increases with s and is independent from a, so does the capital increase with procurement. We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}(r+\delta)\,k$, which implies that $\pi^*(s,a,1) - \pi^*(s,a,0)$ is proportional to the capital increase $k^*(s,a,1) - k^*(s,a,0)$. This could have also be seen by combining equations (E.9) and (E.21), which allows to express $\pi^*(s,a,1) - \pi^*(s,a,0) = \frac{1}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{r+\delta}\right]^{\sigma-1} B_g^\sigma s^{\sigma-1}$.

E.4.2 Constrained firms

For financially constrained firms, the effects of a procurement shock are more intricate, depending on the size of ϕ_g relative to ϕ_p and the net worth of the firm.

Lemma 6 *A procurement shock generates a private sector negative spillover if and only if the procurement shock makes the firm more constrained, that is,* $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$.

Proof: The FOC for optimal k_p given $d=1$ is given by equation (E.1), where recall $\frac{\partial p_p y_p}{\partial k}\frac{1}{u} = \frac{\partial p_p y_p}{\partial k_p}$. The FOC for the optimal choice of k for a firm with $d=0$ is given by the same equation (E.1) when $u=1$. The right-hand side of equation (E.1) increases with λ (see Lemma 1), so more constrained firms have a higher marginal product of capital and a lower level of capital in the private sector. Hence, $k_p(s,a,1) < k(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$. ■

Lemma 7 *A procurement shock generates a private sector negative spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g p_g(s,a,1)\, y_g(s,a,1) < k_g(s,a,1)$

Proof: The demand for capital of constrained firms, with or without procurement, implies

$$\begin{aligned} k_p(s,a,0) - \phi_p p_p(s,a,0)\, y_p(s,a,0) &= \phi_a a \\ k_p(s,a,1) - \phi_p p_p(s,a,1)\, y_p(s,a,1) &= \phi_a a - \left[k_g(s,a,1) - \phi_g p_g(s,a,1)\, y_g(s,a,1)\right] \end{aligned}$$

Importantly, the left-hand side of these equations increases with k_p. To see how, note that the derivative of the left-hand side w.r.t. k_p is equal to $1 - \phi_p \frac{\partial p_p y_p}{\partial k_p} = 1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}$ according to equation (E.1). Now, $\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p} < 1$ according to Lemma 1, so the derivative is positive. Hence, if $k_p(s,a,1) < k_p(s,a,0)$ then $[k_p(s,a,1) - \phi_p p_p(s,a,1) y_p(s,a,1)] < [k_p(s,a,0) - \phi_p p_p(s,a,0) y_p(s,a,0)]$ which requires $\phi_g p_g(s,a,1) y_g(s,a,1) < k_g(s,a,1)$. ■

Proposition 13 *When $\phi_g \leq \phi_p$, a procurement shock for constrained firms generates a private sector negative spillover, that is, $k_p(s,a,1) < k(s,a,0)$, makes the firm more constrained, that is, $\lambda(s,a,1) > \lambda(s,a,0)$, and production in the government sector cannot be self-financed, that is, $\phi_g p_g(s,a,1) y_g(s,a,1) < k_g(s,a,1)$. When $\phi_g > \phi_p$ the same will happen, with the exception of firms with very small net worth for which the opposite will happen.*

Proof: To prove the first part, let's rewrite the borrowing constraint for $d = 0$ firms as

$$1 = \phi_a \frac{a}{k(s,a,0)} + \phi_p \frac{p_p(s,a,0)\, y_p(s,a,0)}{k(s,a,0)} \tag{E.33}$$

and for $d = 1$ firms as

$$\begin{aligned} 1 &= \phi_a \frac{a}{k_p(s,a,1) + k_g(s,a,1)} + \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)} \\ &+ (1 - u(s,a,1)) \left[\phi_g \frac{p_g(s,a,1)\, y_g(s,a,1)}{k_g(s,a,1)} - \phi_p \frac{p_p(s,a,1)\, y_p(s,a,1)}{k_p(s,a,1)} \right] \end{aligned} \tag{E.34}$$

If $\phi_g = \phi_p$, firms with $d = 1$ equalize the average product in the public and private sectors, see equations (E.3) and (E.4), so that the third term in equation (E.34) disappears. In this case, if $k_g(s,a,1) = 0$ then equations (E.33) and (E.34) are identical and $k_p(s,a,1) = k(s,a,0)$. However, because the marginal revenue product in the public sector goes to infinity when $k_g(s,a,1) = 0$, it must be that $k_g(s,a,1) > 0$ and hence comparison of equations (E.33) and (E.34) requires $k_p(s,a,1) < k(s,a,0)$. If $\phi_g < \phi_p$, then the third term in equation (E.34) is negative. This can be easily seen by multiplying both sides of equation (E.3) by ϕ_p and both sides of equation (E.4) by ϕ_g. Then whenever $k_g > 0$ and hence $(1 - u) > 0$, equation (E.34) requires $k_p(s,a,1) < k(s,a,0)$ to hold. The second and third parts of the Proposition come from Lemma 6 and Lemma 7 respectively. Finally, for the case $\phi_g > \phi_p$ the third term in equation (E.34) is positive. If $a = 0$ this requires $k_p(s,a,1) > k(s,a,0)$ for equation (E.34) to hold as the first term in the right-hand side of equation (E.34) disappears. For $a > 0$, the first term in the right-hand side of equation (E.34) reappears and offsets this force. More specifically, as a increases, λ falls by Proposition 10, and thus $\frac{p_g(s,a,1) y_g(s,a,1)}{k_g(s,a,1)}$ decreases. In the limit, if a becomes sufficiently large and exceeds the net worth level $a_g^*(s)$ above which the procurement firm is unconstrained, $\frac{p_g(s,a,1) y_g(s,a,1)}{k_g(s,a,1)}$ falls to the unconstrained level of $\frac{\sigma}{\sigma-1}(r+\delta)$, which is strictly smaller than $\frac{1}{\phi_g}$ by Assumption 1. This means that there exists a cutoff level $\bar{a}_g(s)$ such that if $a \in (\bar{a}_g(s), a_g^*(s))$, then $\phi_g p_g(s,a,1) y_g(s,a,1) < k_g(s,a,1)$. And by Lemma 7, this means that the spillover is negative for a in this interval. ■

Proposition 14 *Whenever $\phi_g \geq \phi_p > 0$, a procurement shock generates an increase in firm size, i.e., $k(s,a,1) > k(s,a,0)$, $\forall a,s$. Whenever $\phi_g < \phi_p$ the opposite may happen. In the particular case of $\phi_g = \phi_p = 0$, a procurement shock does not change firm size.*

Proof: We prove the $\phi_g \geq \phi_p > 0$ case by contradiction by showing that if $k(s,a,1) \leq k(s,a,0)$, then the borrowing constraint for the firm with $d = 1$ would not bind, which could not be optimal; so it must be that $k(s,a,1) > k(s,a,0)$. To se why, we start with the case $k(s,a,1) = k(s,a,0)$. In this situation, the firm with $d = 1$ optimally chooses $u(s,a,1) < 1$ because the marginal revenue product of revenues in the public sector tends to infinity as u tends to 1. This generates more revenues and because $\phi_g \geq \phi_p > 0$, Lemma 4 guarantees that this also generates more (unused) borrowing capacity, so it cannot be optimal. If $k(s,a,1) < k(s,a,0)$ and $u(s,a,1) = 1$ this again generates slack in the borrowing constraint because of Lemma 3, and cannot be optimal. But lowering u generates the same or further slack when $\phi_g \geq \phi_p > 0$, see Lemma 4. So $k(s,a,1) < k(s,a,0)$ cannot be optimal either. Note that the argument by contradiction requires that $\phi_g \geq \phi_p > 0$ such that when the firm with $d = 1$ substitutes private revenues with public revenues the borrowing capacity increases. When $\phi_g < \phi_p$, instead, the contrary happens because selling to the government limits the borrowing capacity of the firm, and the proof does not hold. For example, it can be shown that with $0 = \phi_g < \phi_p$ we will have $k(s,a,1) < k(s,a,0)$. Using the financial constraint, the difference in the capital that can be financed with $d = 1$ and $d = 0$ when $\phi_g = 0$ is given by,

$$k(s,a,1) - k(s,a,0) = \phi_p \left[p_p(s,a,1)\, y_p(s,a,1) - p_p(s,a,0)\, y_p(s,a,0)\right]$$

Proposition 13 says that there is a negative private sector spillover, $p_p(s,a,1)\, y_p(s,a,1) < p_p(s,a,0)\, y_p(s,a,0)$ whenever $\phi_g < \phi_p$, and thus $k(s,a,1) < k(s,a,0)$. Finally, note that with $\phi_g = \phi_p = 0$, $k(s,a,1) = k(s,a,0)$ as constrained firms' k is determined only by a. ■

Proposition 15 *Having access to procurement generates extra profits, that is, $\pi(s,a,1) > \pi(s,a,0)$ $\forall s,a$. Whenever $\phi_g \leq \phi_p$, the value of procurement is increasing in net worth; whenever $\phi_g > \phi_p$, the value of procurement is generally increasing in net worth except for firms with very low net worth when the opposite will happen. The value of procurement is increasing in firm productivity whenever $\phi_g \geq \phi_p$.*

Proof: The first part is trivial. A firm with $d = 1$ has profits equal to

$$\pi(s,a,1) = p_p(s,a,1)\, y_p(s,a,1) + p_g(s,a,1)\, y_g(s,a,1) - (r+\delta)\, k(s,a,1)$$

and can always replicate the profits of a firm with $d = 0$ by choosing $u(s,a,1) = 1$. Because of our functional form assumptions, the marginal revenue product of capital in the public sector, $\partial p_g y_g / \partial k_g$, tends to infinity whenever $u(s,a,1) = 1$, so it means that it is optimal for any firm with $d = 1$ to choose $u(s,a,1) < 1$ and increase profits compared to the case $u(s,a,1) = 1$ and therefore compared to the case of no procurement. For the second part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial a} > 0$. Equations (E.18) and (E.27) imply

$$\frac{\partial\left[\pi(s,a,1) - \pi(s,a,0)\right]}{\partial a} = \phi_a\left[\lambda(s,a,1) - \lambda(s,a,0)\right] > 0$$

and the sign of $\lambda(s,a,1) - \lambda(s,a,0)$ is given by Proposition 13. Finally, for the third part we want to show that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$ whenever $\phi_g \geq \phi_p$. (E.19) and (E.28) imply

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = (1 + \phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s} + (1 + \phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s} - (1 + \phi_p\lambda(s,a,0))\frac{\partial p_p y_p}{\partial s}$$

Note that $\frac{\partial p_p y_p}{\partial s} = \frac{k_p}{s}\frac{\partial p_p y_p}{\partial k_p} = \frac{k_p}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for the public good. Substituting these expressions in the above equation gives

$$\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} = \frac{r+\delta+\lambda(s,a,1)}{s}\left[k_p(s,a,1) + k_g(s,a,1)\right] - \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$$

With $\phi_g \geq \phi_p$, Proposition 14 states that $k_p(s,a,1) + k_g(s,a,1) > k_p(s,a,0)$. Therefore, whenever $\lambda(s,a,1) > \lambda(s,a,0)$ we can guarantee that $\frac{[\partial\pi(s,a,1) - \partial\pi(s,a,0)]}{\partial s} > 0$. According to Proposition 13 this will generally happen, except for very low a when $\lambda(s,a,1) < \lambda(s,a,0)$. However, in this case we can still show the statement to be true by showing that $\frac{r+\delta+\lambda(s,a,1)}{s}k_p(s,a,1) > \frac{r+\delta+\lambda(s,a,0)}{s}k_p(s,a,0)$. To show this, we take the FOC for k_p in equation (E.1) to obtain an expression $\lambda = \frac{\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}$. Then adding $(r+\delta)$ in both sides, rearranging, and multiplying by k_p in both sides we obtain

$$(r+\delta+\lambda)k_p = \left[1 - \phi_p(r+\delta)\right]\frac{\frac{\partial p_p y_p}{\partial k_p}}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p$$

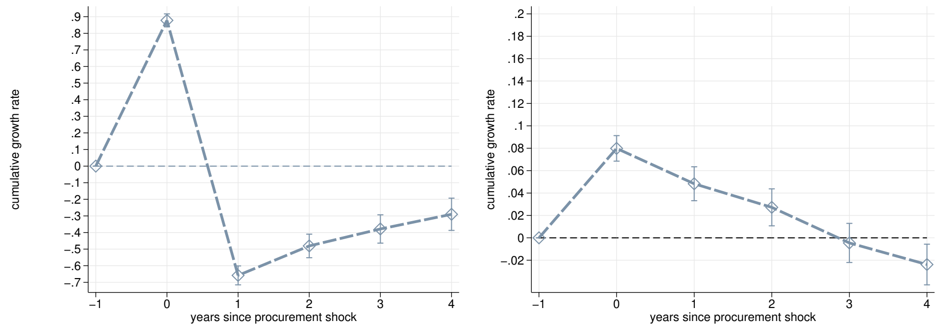
Using our functional form for the revenue function, we can rewrite the last terms as:

$$\frac{\frac{\partial p_p y_p}{\partial k_p}}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p = \frac{\sigma-1}{\sigma}\frac{B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}}{1 - \phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}}$$

Taking the derivative of this object w.r.t. k_p, we have:

$$\begin{aligned}
\frac{\partial}{\partial k_p}\left(\frac{\frac{\partial p_p y_p}{\partial k_p}}{1 - \phi_p\frac{\partial p_p y_p}{\partial k_p}}k_p\right) &\propto \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left[1 - \phi_p\frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right] - B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}\frac{1}{\sigma}\phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}-1} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1 - \phi_p B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\right\} \\
&= \frac{\sigma-1}{\sigma}B_p s^{\frac{\sigma-1}{\sigma}}k_p^{\frac{-1}{\sigma}}\left\{1 - \phi_p\frac{p_p y_p}{k_p}\right\} > 0
\end{aligned}$$

where the last inequality follows from the fact that by Lemma 3, $\phi_p \frac{p_p y_p}{k_p} < 1$ for constrained firms. This establishes that for constrained firms, the term $(r+\delta+\lambda)k_p$ must be higher whenever k_p is higher. Therefore, if $\lambda(s,a,1) < \lambda(s,a,0)$, the k_p FOC, implies that $k_p(s,a,1) > k_p(s,a,0)$, which in turns implies $[r+\delta+\lambda(s,a,1)]k_p(s,a,1) > [r+\delta+\lambda(s,a,0)]k_p(s,a,0)$. And trivially, this implies $\frac{r+\delta+\lambda(s,a,1)}{s}\left[k_p(s,a,1) + k_g(s,a,1)\right] - \frac{r+\delta+\lambda(s,a,0)}{s} k_p(s,a,0)$, proving the statement whenever $\phi_g \geq \phi_p$ and $\lambda(s,a,1) < \lambda(s,a,0)$. ■



(a) λ **(b)** b

Notes: This figure shows the cumulative estimated impact (β_1^h) of obtaining a procurement contract for horizons $h = 0, 1, 2, 3, 4$. The left panel shows the effects on λ. The right panel shows the effects on b.

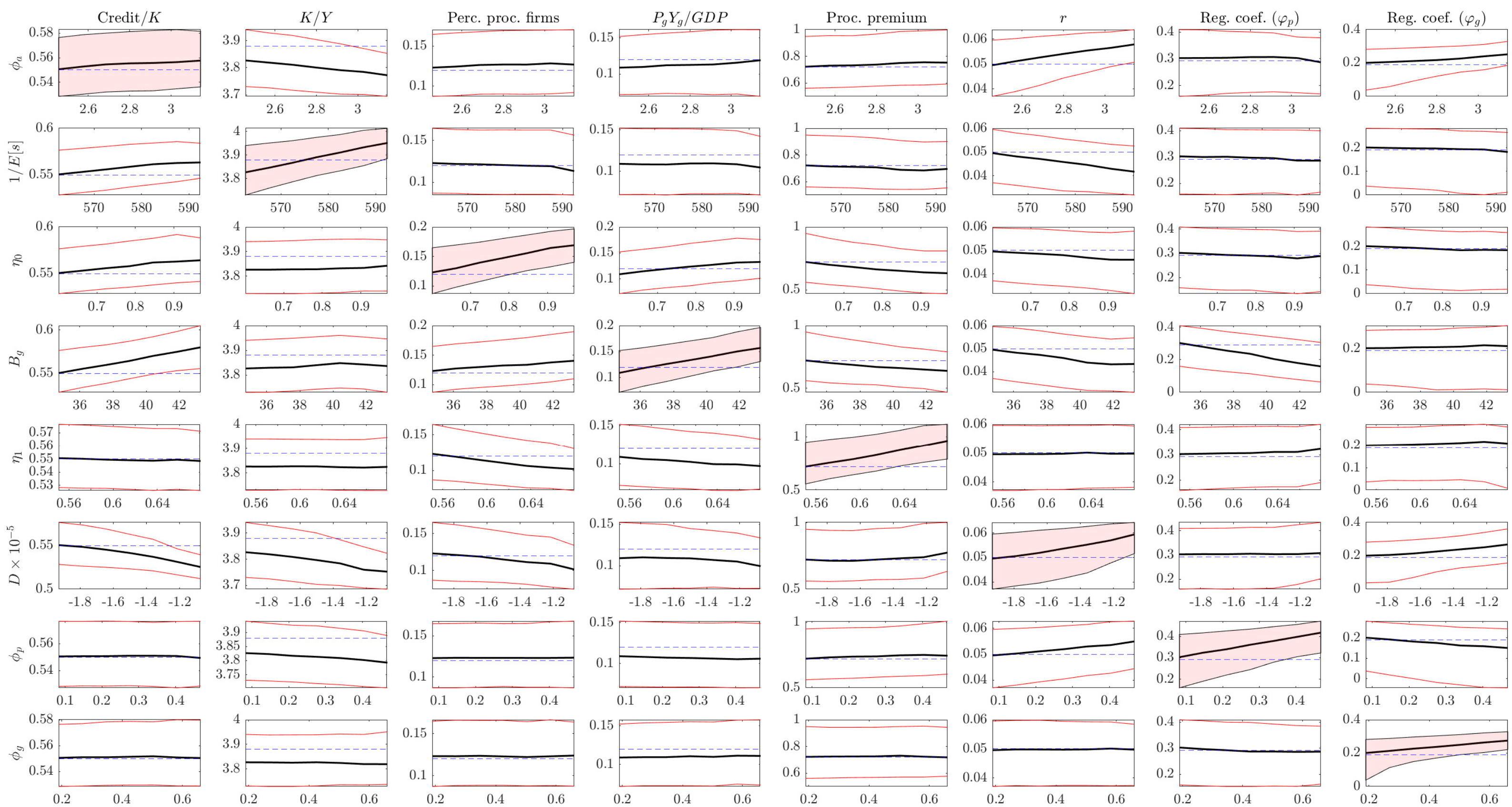
Figure A.V. Treatment effects of procurement in the baseline calibration

F Identification of model parameters

Our strategy consists of internally calibrating 8 parameters so that the model matches 8 moments. We want to show that our choice of targets is justified by the fact that each of these targets is "particularly informative" of a particular parameter.

First, we draw many parameter combinations based on a 8-dimensional hypercube. Second, we solve for the model's steady state and calculate the relevant moments for all parameter combinations. And third, we plot how the 25th percentile, the 50th, and the 75th percentile of a given moment changes as we move along the vector of its associated parameter. Intuitively, this figure shows how a particular moment is affected by a specific parameter letting other parameters move. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification. In Figure A.VI, we show that each moment is especially informative of one parameter.

Figure A.VI. Identification of the model parameters



Notes: This figure plots the relationship between percentiles of a given moment and a specific parameter. The thick black line refers to the median, and the thiner black lines above and below refer to the 85th and 15th percentile respectively. The dashed blue line represents the targeted value of the moment. These lines have been generated by solving the steady-state of the model using combinations of parameters drawn from a 8-dimensional hypercube.

G Further details on treatment effects in benchmark

To analyze dynamic treatment effects of procurement in our model and in the data, we estimate the same local projection panel regressions as in Section 3.1. See Figure A.V.

H Aggregate output, productivity, and prices

GDP and aggregate TFP. We can define GDP in the model as

$$Y \equiv Y_p + P_g Y_g = \text{TFP}_p K_p + P_g \text{TFP}_g K_g = \text{TFP} K \tag{H.1}$$

where $K \equiv K_p + K_g$ and aggregate TFP in units of the private goods is defined by,

$$\text{TFP} \equiv \frac{K_p}{K}\text{TFP}_p + \frac{K_g}{K} P_g \text{TFP}_g \tag{H.2}$$

Sectorial TFP. The TFP for the private and public sectors are given by,

$$\text{TFP}_p \equiv \frac{Y_p}{K_p} = \left[\int_{[0,1]} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}}, \quad \text{TFP}_g \equiv \frac{Y_g}{K_g} = \left[\int_{I_g} \frac{1}{m_g}\left(s_i \frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g - 1}} \tag{H.3}$$

$$\text{where} \quad \frac{1}{\overline{\text{MRPK}}_p} \equiv \int_{[0,1]} \frac{p_{ip} y_{ip}}{P_p Y_p} \frac{1}{\text{MRPK}_{ip}} di, \quad \frac{1}{\overline{\text{MRPK}}_g} \equiv \int_{I_g} \frac{1}{m_g} \frac{p_{ig} y_{ig}}{P_g Y_g} \frac{1}{\text{MRPK}_{ig}} di \tag{H.4}$$

Absent financial frictions there would be no heterogeneity in $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\text{TFP}_p^* = \left[\int_{[0,1]} s_i^{\sigma_p - 1} di\right]^{\frac{1}{\sigma_p - 1}} \quad \text{and} \quad \text{TFP}_g^* = \left[\int_{I_g} \frac{1}{m_g} s_i^{\sigma_g - 1} di\right]^{\frac{1}{\sigma_g - 1}} \tag{H.5}$$

Relative price of public sector good. Using the definitions of P_g and P_p , the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} p_{ip}^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i}\text{MRPK}_{ig}\right)^{1-\sigma_g} di\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i}\text{MRPK}_{ip}\right)^{1-\sigma_p} di\right]^{\frac{1}{1-\sigma_p}}}$$

which follows from the definition of MRPK_{ip}, and the production function as,

$$\text{MRPK}_{ip} \equiv \frac{\partial p_{ip} y_{ip}}{\partial k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} \frac{p_{ip} y_{ip}}{k_{ip}} = \frac{\sigma_p - 1}{\sigma_p} p_{ip} s_i \quad \Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p - 1} \frac{1}{s_i} \text{MRPK}_{ip}$$

and the same applies for MRPK_{ig}. Next multiplying and dividing by $\overline{\text{MRPK}}_g$ in the numerator and by $\overline{\text{MRPK}}_p$ in the denominator we obtain,

$$\frac{P_g}{P_p} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_g}{\text{MRPK}_{ig}}\right)^{1-\sigma_g} di\right]^{\frac{1}{\sigma_g - 1}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i} \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}}\right)^{\sigma_p - 1} di\right]^{\frac{1}{1-\sigma_p}}} = \frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p} \frac{\text{TFP}_p}{\text{TFP}_g}$$

Relative sectoral TFP. Given the definition of TFP_p in equation (H.3), we can write

$$\begin{aligned}
\text{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}} \right)^{\sigma_p - 1} di + (1 - m_g) \int_{I_g^c} \frac{1}{1 - m_g} \left(s_i \frac{\overline{\text{MRPK}}_p}{\text{MRPK}_{ip}} \right)^{\sigma_p - 1} di \right]^{\frac{1}{\sigma_p - 1}} \\
&= \left[m_g \text{TFP}_{p,I_g}^{\sigma_p - 1} + (1 - m_g) \text{TFP}_{p,I_g^c}^{\sigma_p - 1} \right]^{\frac{1}{\sigma_p - 1}}
\end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\text{TFP}_p/\text{TFP}_g$:

$$\frac{\text{TFP}_p}{\text{TFP}_g} = \left[m_g \left(\frac{\text{TFP}_{p,I_g}}{\text{TFP}_g} \right)^{\sigma_p - 1} + (1 - m_g) \left(\frac{\text{TFP}_{p,I_g^c}}{\text{TFP}_g} \right)^{\sigma_p - 1} \right]^{\frac{1}{\sigma_p - 1}} \tag{H.6}$$

The first term in equation (H.6) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \; \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\text{TFP}_{p,I_g}/\text{TFP}_g$ fall. The second term in equation (H.6) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\text{TFP}_p/\text{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\text{TFP}_p/\text{TFP}_g < 1$.

Relative sectoral $\overline{\textbf{MRPK}}$. Given the definition of $\overline{\text{MRPK}}_p$ in (H.4), we can write

$$\begin{aligned}
\overline{\text{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_p Y_p} \int_{I_g} \frac{p_{ip} y_{ip}}{R_{p,I_g}} \text{MRPK}_{ip}^{-1} di + \frac{R_{p,I_g^c}}{P_p Y_p} \int_{I_g^c} \frac{p_{ip} y_{ip}}{R_{p,I_g^c}} \text{MRPK}_{ip}^{-1} di \right]^{-1} \\
&= \left[\frac{R_{p,I_g}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g^c}^{-1} \right]^{-1}
\end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\text{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p/\overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g} \right)^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g} \right)^{-1} \right]^{-1} \tag{H.7}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The first term in equation (H.7) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g} > \overline{\text{MRPK}}_g$. The second term in equation (H.7) reflects both between-firm misallocation and selection into procurement.

I Decreasing Returns to Scale

We characterize the crowding-out effect of procurement to private sales for unconstrained firms under decreasing returns to scale. We also present the result of running our Counterfactual 1 in an economy in which there are DRS. The firm solves the problem:

$$\pi = \max_{k,y_p,y_g} \left\{ B_p^{\frac{1}{\sigma}} y_p^{\frac{\sigma-1}{\sigma}} + B_g^{\frac{1}{\sigma}} y_g^{\frac{\sigma-1}{\sigma}} - (r+\delta)k \right\} \tag{I.1}$$

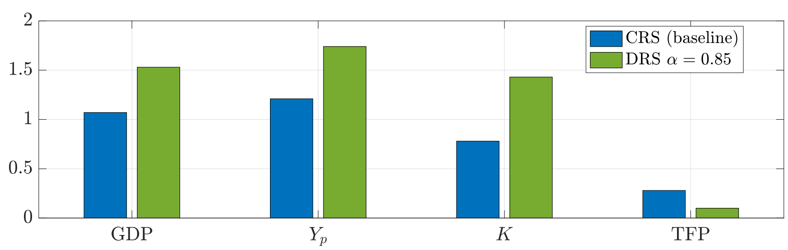
$$\text{s.t.} \quad y_p + y_g \leq s\ k^{\alpha} \tag{I.2}$$

The optimal solution of the firm implies a private sector revenue given by:

$$p_p y_p = B_p^{\frac{1}{\sigma}} y_p^{\frac{\sigma-1}{\sigma}} = B_p (B_g + B_p)^{\frac{(\sigma-1)(\alpha-1)}{\sigma-\alpha(\sigma-1)}} \left[\left(\frac{\sigma-1}{\sigma} \right) \frac{\alpha}{r+\delta} \right]^{\frac{\alpha(\sigma-1)}{\sigma-\alpha(\sigma-1)}} s^{\frac{\sigma-1}{\sigma-\alpha(\sigma-1)}} \tag{I.3}$$

The elasticity of $p_p y_p^*$ with respect to B_g is given by $\frac{\partial\ log\ p_p y_p^*}{\partial\ log\ B_g} = \left(\frac{(\sigma-1)(\alpha-1)}{\sigma-\alpha(\sigma-1)}\right)\left(\frac{1}{B_p+B_g}\right) < 0$.

Figure A.VII. Counterfactual 1: CRS vs. DRS



Notes: This figure shows the effects of running Counterfactual 1 both in our CRS (baseline) economy and a recalibrated economy with DRS ($\alpha = 0.85$). The y-axis shows the percentage changes relative to the benchmark economy.