Country Size, International Trade, and Aggregate Fluctuations in Granular Economies

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This paper proposes a new mechanism by which country size and international trade affect macroeconomic volatility. We study a model with heterogeneous firms that are subject to idiosyncratic firm-specific shocks, calibrated to data for the 50 largest economies in the world. When the firm size distribution follows a power law with an exponent close to minus one, idiosyncratic shocks to large firms have an impact on aggregate volatility. Smaller countries have fewer firms and, thus, higher volatility. Trade opening makes the large firms more important, thus raising macroeconomic volatility. Trade can increase aggregate volatility by 15–20 percent in some small open economies.

I. Introduction

Output volatility varies substantially across economies: over the past 35 years, the standard deviation of annual real per capita GDP growth has been two and a half times higher in non-OECD countries compared to

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the OECD countries. Understanding the sources of these differences is important, as aggregate volatility itself has an impact on a wide variety of economic outcomes.[1]

This paper investigates the role of large firms in explaining cross-country differences in aggregate volatility. We show that the impact of shocks to large firms on aggregate volatility can help account for two robust empirical regularities: (i) smaller countries are more volatile, and (ii) more open countries are more volatile. The key ingredient of our study is that the distribution of firm size is very fat tailed: the typical economy is dominated by a few very large firms (Axtell 2001). In a recent contribution, Gabaix (2011) demonstrates that under these conditions idiosyncratic shocks to individual firms do not cancel out and can instead generate aggregate fluctuations (see also Delli Gatti et al. 2005). Gabaix provides both statistical and anecdotal evidence that even in the largest and most diversified economy in the world—the United States—shocks to the biggest firms can appreciably affect macroeconomic fluctuations. The economy is "granular" rather than smooth.

We develop a theoretical and quantitative framework to study the consequences of this phenomenon in a large cross section of countries. The analysis is based on the canonical multicountry model with heterogeneous firms in the spirit of Melitz (2003) and Eaton, Kortum, and Kramarz (2011), implemented on the 50 largest economies in the world. In order to study the impact of large firms on aggregate fluctuations, the equilibrium total number of firms is determined endogenously in the model, and the parameters are calibrated to match the observed firm size distribution. The solution procedure targets the key aggregate country characteristics—GDPs and average trade volumes—and successfully reproduces a number of nontargeted features of the micro data, such as the share of firms that export and the relative size of the largest firms across countries.

Our main results can be summarized as follows. First, the model endogenously generates a negative relationship between country size and aggregate volatility. The reason is that smaller countries will have a smaller equilibrium number of firms (as implied by many models since at least Krugman [1980]), and thus shocks to the largest firms will matter more for aggregate volatility. In effect, smaller economies are less diversified when diversification is measured at the firm level. The model matches this relationship not only qualitatively but also quantitatively: the rate at

not be attributed to the International Monetary Fund, its Executive Board, or its management. The Supplementary Web Appendix to this paper is available at http://alevchenko.com/diGiovanni_Levchenko_granular_web_appendix.pdf.

[1] Numerous studies identify its effects on long-run growth (Ramey and Ramey 1995), welfare (Pallage and Robe 2003; Barlevy 2004), and inequality and poverty (Gavin and Hausmann 1998; Laursen and Mahajan 2005).

which volatility decreases in country size in the model is very similar to what is observed in the data. Both in the model and in the data, a typical country that accounts for 0.5 percent of world GDP (such as Poland or South Africa) has aggregate volatility that is two times higher than that of the largest economy in the world—the United States.

Second, trade openness increases volatility by making the economy more granular. When a country opens to trade, only the largest and most productive firms export, while smaller firms shrink or disappear (Melitz 2003). This effect implies that after opening, the biggest firms become even larger relative to the size of the economy, thus contributing more to aggregate output fluctuations. In the counterfactual exercise, we compute what aggregate volatility would be for each country in autarky and compare it to the volatility under the current trade costs. It turns out that at the levels of trade openness observed today, international trade increases volatility relative to autarky in every country. The importance of trade for aggregate volatility varies greatly depending on country characteristics. In the largest economies such as Japan or the United States, aggregate volatility is only 1.5–3.5 percent higher than it would have been in complete autarky. In small, remote economies such as South Africa or New Zealand, trade raises volatility by about 10 percent compared to autarky. Finally, in small, highly integrated economies such as Denmark or Romania, international trade raises aggregate volatility by some 15–20 percent.

The theoretical link between country size, trade openness, and volatility we explore in this paper has not previously been proposed. Head (1995) and Crucini (1997) examine the relationship between country size and volatility in a two-country international real business cycle (IRBC) model. In those papers, the smaller country has higher volatility because the world interest rate is less sensitive to shocks occurring in that country. Thus, following a positive shock, it can expand investment without much of an impact on interest rates.[2] Our explanation for the size-volatility relationship is qualitatively different and relies instead on the notion that smaller countries have fewer firms. When it comes to the relationship between trade openness and volatility, existing explanations have focused on the propagation of global demand or supply shocks (Newbery and Stiglitz 1984; Kraay and Ventura 2007). We show that trade can increase volatility even if the nature of shocks affecting the firms is

[2] The Supplementary Web Appendix implements the canonical IRBC model of Backus, Kehoe, and Kydland (1995) and examines the relationship between country size and volatility, and between trade openness and volatility, in that model. It turns out that while the calibrated IRBC model can produce higher volatility in smaller countries, the relationship between country size and volatility in that model is two orders of magnitude flatter than what is observed in the data. The relationship between trade openness and volatility in the IRBC model is ambiguous, its sign depending crucially on the elasticity of substitution between domestic and foreign goods.

unchanged upon opening. Finally, the mechanism in our model resembles the traditional arguments that smaller countries, and more open countries, will have a less diversified sectoral production structure and thus exhibit higher volatility (see Katzenstein [1985], OECD [2006], and Blattman, Hwang, and Williamson [2007], among many others). Our analysis shows that this argument applies to individual firms as well as to sectors and makes this point quantitatively precise by calibrating the model to the observed firm size distribution.

Our work is also related to the empirical literature that studies macroeconomic volatility using disaggregated data. Koren and Tenreyro (2007) explore the importance of sector-specific shocks in explaining the relationship between a country's level of development and its aggregate volatility, while di Giovanni and Levchenko (2009, 2012) use sector-level data to study the openness-volatility relationship. Canals et al. (2007) analyze sector- and firm-level export data and demonstrate that exports are highly undiversified across both firms and sectors and across destinations. Furthermore, they show that this feature of export baskets can explain why aggregate macroeconomic variables cannot account for much of the movements in the current account.[3]

The rest of the paper is organized as follows. Section II presents the empirical regularities that motivate our study, as well as some novel stylized facts about firm size distributions in a large cross section of countries. Section III develops a simple theoretical framework and illustrates analytically the mechanisms behind the key results of the paper. Section IV presents the quantitative results based on a calibrated model of the world economy. Section V discusses robustness checks and results based on model perturbations. Section VI presents conclusions.

II. Basic Facts

Figure 1 presents a scatter plot of log macroeconomic volatility against log country size for a sample of 143 countries. Volatility is the standard deviation of the yearly growth rate of real per capita GDP, while country size is the share of the country in world GDP.[4] The figure depicts the

[3] Our work is complementary to the research agenda that studies the impact of firm dynamics on macroeconomic outcomes in two-country IRBC models. Ghironi and Melitz (2005) use the heterogeneous firms model to help account for the persistence of deviations from purchasing power parity (PPP), while Alessandria and Choi (2007) and Ruhl (2008) evaluate the quantitative importance of firm entry and exit for aggregate trade dynamics. An important difference between these papers and our work is that these contributions examine consequences of aggregate shocks, while in our paper all the shocks are at the firm level. In addition, our work features multiple countries and explains cross-sectional differences in volatility between countries.

[4] Detailed variable definitions and sources for all the data in this section are described in App. A.

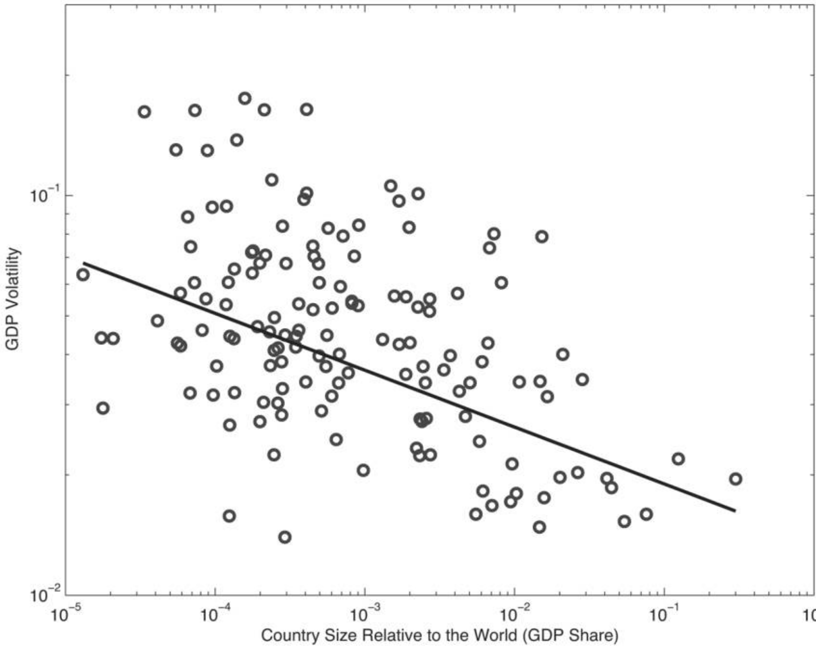

FIG. 1.—Country size and aggregate volatility. This figure reports the partial correlation plot of aggregate volatility, measured as the standard deviation of the annual growth rate of per capita GDP over 1970–2006, on the *y*-axis against country size on the *x*-axis, after netting out the impact of per capita income. Both axes are in log scale. Source: World Bank World Development Indicators.

partial correlation between these two variables after netting out per capita income, as it has been shown that high-income countries tend to experience lower volatility. As established by Canning et al. (1998) and Furceri and Karras (2007), among others, smaller countries are more volatile. The elasticity of volatility with respect to country size is about −0.14 in this set of countries, and the relationship is highly significant, with a *t*-statistic of 6.2.

At the same time, it has been argued that countries that trade more tend to be more volatile. This empirical regularity has been demonstrated in a cross section of countries by Easterly, Islam, and Stiglitz (2001) and Kose, Prasad, and Terrones (2003). The cross-country evidence is likely to be affected by reverse causality and omitted variables problems. In addition, in a cross section of countries one does not typically have enough power to distinguish between trade openness and other correlates of macroeconomic volatility (such as, most relevant here, country size). Di Giovanni and Levchenko (2009) investigate the openness-volatility relationship in great detail using industry-level data, which makes it possible to overcome these econometric estimation concerns. They confirm that the relationship between trade openness and macroeconomic volatility is indeed positive and economically significant, even after controlling for the size of both countries and sectors using fixed effects. Below we follow an alternative and complementary strategy to isolate the impact of openness on volatility. We develop a quantitative model that captures a particular channel for this relationship and evaluate the impact of trade using counterfactual scenarios.

The analysis below focuses on the role of large firms in explaining these cross-country patterns. Anecdotal evidence on the importance of large firms for aggregate fluctuations abounds. Here, we describe two examples in which the roles of country size and international trade are especially evident. In New Zealand a single firm, Fonterra, is responsible for a full one-third of global dairy exports (it is the world's single-largest exporter of dairy products). Such a large exporter from such a small country clearly matters for the macroeconomy. Indeed, Fonterra's sales (95 percent of which are exports) account for 20 percent of New Zealand's overall exports and 7 percent of its GDP.[5] Two points about this firm are worth noting. First, international trade clearly plays a prominent role in making Fonterra as large as it is. And second, the distribution of firm size in the dairy sector is indeed highly skewed. The second-largest producer of dairy products in New Zealand is 1.3 percent the size of Fonterra. This phenomenon is not confined to commodity-exporting countries. In Korea, a larger manufacturing-based economy, the 10 biggest business groups account for 54 percent of GDP and 51 percent of total exports. Even among the top 10, the distribution of firm size and total exports is extremely skewed. The largest one, Samsung, is responsible for 23 percent of exports and 14 percent of GDP (see fig. 2).[6]

There is a growing body of additional evidence on the predominance of the largest firms, especially in total exports. Eaton, Kortum, and Sotelo (2012) report that in France the top 10 exporters account for nearly 25 percent of total exports. According to Canals et al. (2007), in Japan the top 10 exporters account for about 30 percent and the top 50 exporters for more than half of total exports. Cebeci et al. (2012) report that in a sample of 45 developed and developing countries, the top 1 percent of exporters account for 55 percent of total exports on average. In a number of countries (Chile, Peru, South Africa) the top 1 percent of exporters are responsible for 80 percent or so of total exports.

[5] It is important to note that GDP represents value added, and thus Fonterra's total sales are less than 7 percent of the total sales of all firms in New Zealand. However, because exports are recorded as total sales, Fonterra's export sales are directly comparable to New Zealand's total exports. The same caveat applies to the example that follows.

[6] It turns out that the size distribution of firms is quite skewed even within business groups. For instance, breaking Samsung down into its constituent firms reveals that the sales of Samsung Electronics alone accounted for 7 percent of GDP and 15.5 percent of Korea's exports in 2006.

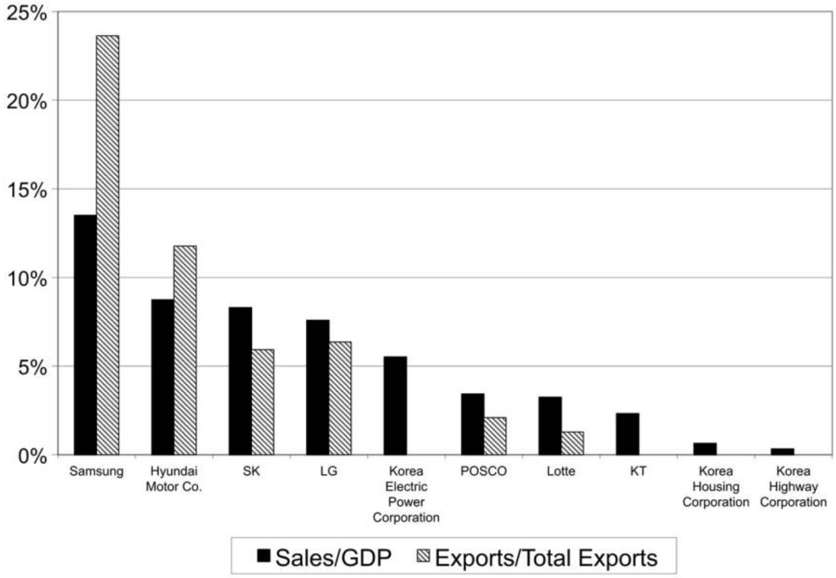

FIG. 2.—Korean business groups' sales as a share of GDP and total exports. This figure reports the 2006 sales of the top 10 Korean business groups, as a share of Korean GDP (dark bars) and total Korean exports (light bars). Source: Korean Development Institute.

Gabaix (2011) shows that aggregate volatility due to the idiosyncratic shocks to firms is an increasing function of the Herfindahl index of the firms' output shares. To produce the country size-volatility relationship in figure 1 through the shocks to large firms, it must be the case that smaller countries have higher Herfindahl indices of firm output: they are less diversified. Figure 3*A* presents the partial correlations between the Herfindahl index of firm sales and country size, after netting out the impact of per capita income, with all variables in natural logs.[7] The figure also plots the ordinary least squares best fit through the data, along with the slope coefficients, standard errors, and the R^2's. The firm-level data used to compute the Herfindahl indices come from the ORBIS database described in Appendix A. Because the number of firms covered by ORBIS varies substantially across countries, we present the results for three samples: (i) all 134 countries for which it is possible to calculate the Herfindahl index in ORBIS data, (ii) the 81 countries with sales data for at least 100 firms, and (iii) the 52 countries with sales data for at least

[7] The Herfindahl index is defined as the sum of squared shares of firm sales in total sales, $h = \sum_k h(k)^2$, where k indexes firms, and $h(k)$ is the share of firm k in total sales by all firms.

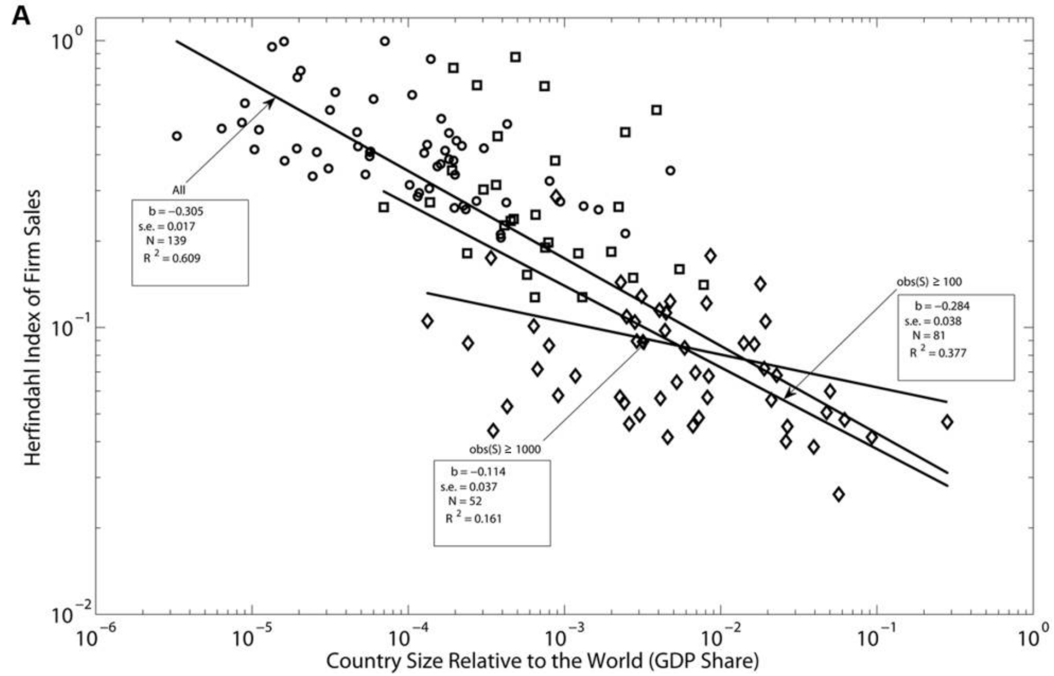

FIG. 3.—Country size, firm sales concentration, and size of large firms. These figures present the scatter plots of log country size and (*A*) the log Herfindahl index of firm sales, (*B*) the log size of the 10 largest firms, and (*C*) the log size of the largest firm, in all cases after netting out per capita GDP. The countries with more than 1,000 firms with sales data are labeled with diamonds, the countries with between 100 and 1,000 firms with sales data are labeled with squares, and the countries with fewer than 100 firms with sales data are labeled with circles. The regression lines through the samples of (i) all countries, (ii) countries with ≥100 firms, and (iii) countries with ≥1,000 firms are plotted through the data. Both axes are in log scale. Sources: ORBIS and World Bank World Development Indicators.

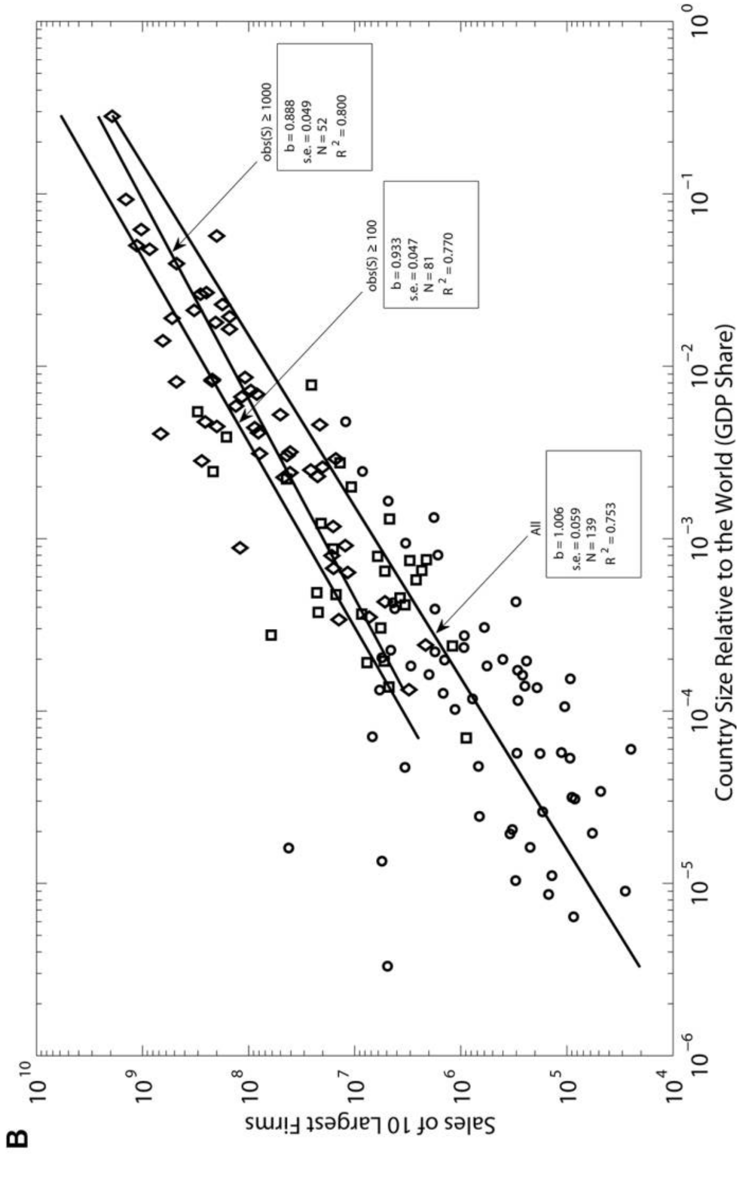

FIG. 3.—(*Continued*)

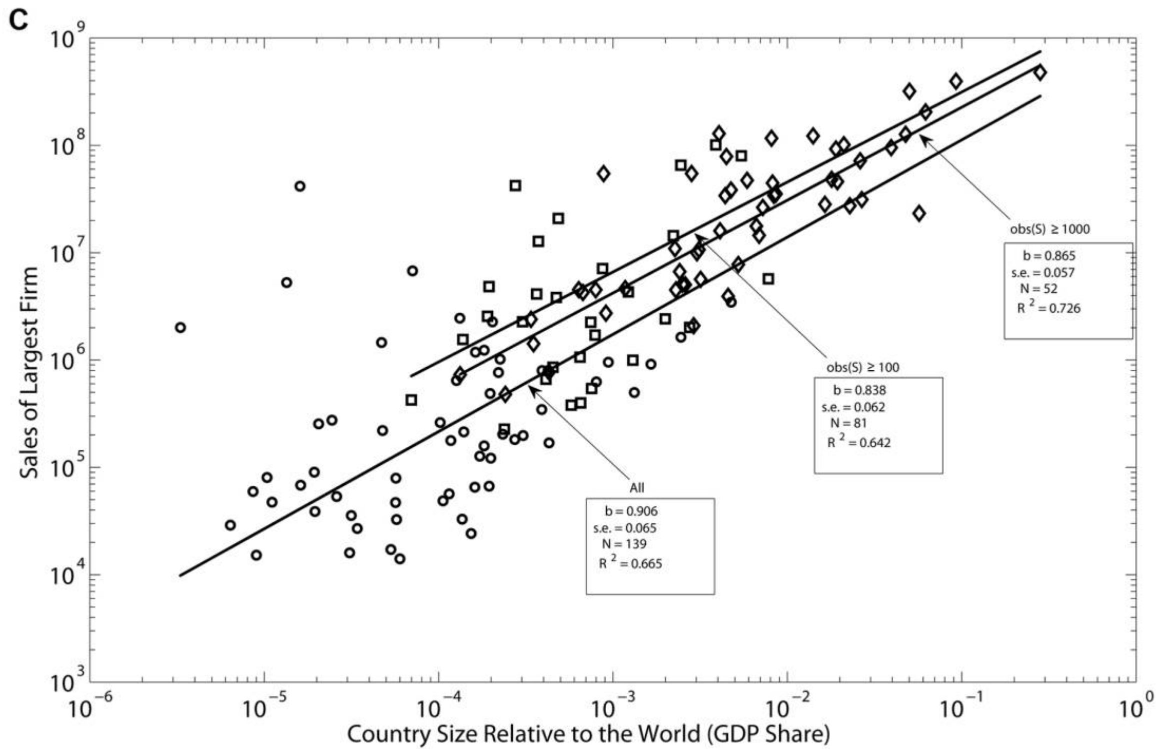

FIG. 3.—(*Continued*)

1,000 firms. The countries with different numbers of firms are labeled with different symbols. Figure 3*A* shows that the relationship between the Herfindahl index and country size is negative as expected and highly statistically significant in all three samples.

The Herfindahl index is the variable most directly relevant to the quantitative results in the paper. However, because ideally it requires information on the entire firm size distribution, the Herfindahl index may also be most heavily influenced by differences in coverage in the ORBIS database. Because of this, we also present the relationship of country size to two other indicators of firm size: the combined sales of the 10 largest firms in the country and the size of the single-largest firm. These indicators focus on the very largest firms that are measured more reliably in the data, and thus the problems of coverage are less severe. In addition, these empirical relationships capture a related feature of the data that is crucial for evaluating the role of large firms for the country size–volatility relationship. A bigger country could either have larger firms or have more firms than a smaller country. There are two extreme cases that can be useful benchmarks. The first is that the largest firms in big countries are no bigger than the large firms in small countries. That is, what makes a country larger is that it has more firms, not bigger firms. This hypothetical possibility would manifest itself as a slope of zero in figures 3*B* and 3*C* and would have important implications for the relationship between country size and (granular) volatility. In particular, a slope of zero would imply that volatility declines very rapidly in country size. The other benchmark is a slope of one. When the slope is one, the largest firms are no bigger relative to the size of the economy in small countries compared to large ones. That would suggest that larger countries have larger firms but not more firms and that the size-volatility relationship is perfectly flat: (granular) volatility in the larger countries is no lower than in small countries.[8] Since these two possibilities have very different implications for how aggregate volatility changes with country size, it is important for our model to match the relative size of the largest firms in countries of different sizes.

Figure 3*B* depicts the partial correlation between the log size of the 10 largest firms and log country size, once again after netting out log per capita income. The results are reported for all three ORBIS samples, as above. There is a significant positive relationship between the absolute size of the largest 10 firms and country size: not surprisingly, larger countries have bigger firms, with an elasticity of around 0.9. The slope coefficient and the fit are both quite stable across the samples. Figure 3*C* reports the analogous relationship for the size of the single-largest firm

[8] This discussion is of course only suggestive because the precise outcomes will depend on the details of the rest of the firm size distribution and because we do not have closed-form results with respect to the size of the top one or 10 largest firms.

in each country, with quite similar conclusions. The fact that the slope is much higher than zero, and indeed only slightly below one, is suggestive that volatility will decrease with country size, but at a much slower rate than the zero slope benchmark. As we discuss in Section IV.C, our model matches these slope coefficients quite closely.

III. Theoretical Framework

This section lays out a simplified analytical framework to illustrate the main mechanisms behind the results. The quantitative investigation based on an extended model follows in Section IV. The full description of the equations defining the extended model is presented in Appendix B.

The minimalist framework that can be used to model the role of firms in the relationship between country size and volatility has to feature (i) a firm size distribution that can be matched to the data, (ii) endogenous determination of the set of firms operating in equilibrium, and (iii) idiosyncratic shocks to firms. In addition, to investigate the role of international trade, the model must have (iv) an export participation decision by firms.

A. *The Environment*

Consider a model in the spirit of Melitz (2003), but with a discrete number of goods as in Krugman (1980). The world is made up of $\mathcal{C}$ countries, indexed by $i, j = 1, \ldots, \mathcal{C}$. In country i, buyers (who could be final consumers or firms purchasing intermediate inputs) maximize a standard constant elasticity of substitution (CES) objective over the set of J_i varieties available in country i. It is well known that in country i demand for an individual variety k is equal to

$$x_i(k) = \frac{X_i}{P_i^{1-\varepsilon}} p_i(k)^{1-\varepsilon}, \tag{1}$$

where ε is the elasticity of substitution between varieties, $x_i(k)$ is the total expenditure on good k at price $p_i(k)$, X_i is total expenditure in the economy, and P_i is the ideal price index:

$$P_i = \left[\sum_{k=1}^{J_i} p_i(k)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{2}$$

There is one factor of production, labor, with country endowments given by L_j, $j = 1, \ldots, \mathcal{C}$, and wages denoted by w_j. Production uses both labor and intermediate inputs. In particular, a firm with unit input requirement a must use a input bundles to produce one unit of output. An input bundle in country j has a cost

$$c_j = w_j^{\beta} P_j^{1-\beta}.$$

There are both fixed and variable costs of production and trade. The timing in the economy is depicted in figure 4. At the beginning of the period each potential producer of variety $k = 1, \ldots, \bar{I}_j$ in each $j = 1, \ldots, \mathcal{C}$ must pay an "exploration" cost f_e in order to become an entrepreneur. Upon paying this cost, entrepreneur k discovers her productivity, indexed by a unit input requirement $a(k)$, and faces downward-sloping demand for her unique variety given by (1). On the basis of this draw, each entrepreneur in country j decides whether to operate and which markets to serve. To start serving market i from country j, a firm must pay a fixed cost f_{ij} and an iceberg per-unit cost of $\tau_{ij} > 1$ (with τ_{jj} normalized to one). Having paid the fixed costs of entering these markets, the firm learns the realization of a transitory shock $z(k)$, independently and identically distributed (i.i.d.) across firms. Once all of the uncertainty has been realized, each firm produces with a unit input requirement $a(k)z(k)$, markets clear, and consumption takes place.[9]

If a firm from country j decides to sell to country i, its profit-maximizing price is a constant markup over marginal cost

$$p_i(k) = \frac{\varepsilon}{\varepsilon - 1} \tau_{ij} c_j a(k) z(k),$$

the quantity supplied is equal to

$$\frac{X_i}{P_i^{1-\varepsilon}} \left[\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} c_j a(k) z(k) \right]^{-\varepsilon},$$

and its expected profits from serving i are

$$\mathrm{E}\left[\frac{X_i}{\varepsilon P_i^{1-\varepsilon}} \left[\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} c_j a(k) z(k) \right]^{1-\varepsilon} - c_j f_{ij} \middle| a(k) \right]. \tag{3}$$

[9] Note that the assumption on the timing of events, namely, that the decision to enter markets takes place before $z(k)$ is realized, implies that the realization of the firm-specific transitory shock does not affect the equilibrium number of firms in each market. This simplification lets us analyze the equilibrium production allocation as an approximation around a case in which the variance of z is zero. That is, we abstract from the extensive margin of exports and entry and exit of firms in response to transitory shocks. This simplification delivers substantial analytical convenience, while it is unlikely to affect the results. The reason is that the focus of the paper is on the role of the largest firms in generating aggregate volatility, and the largest firms are inframarginal: their entry decision will be unaffected by the realization of the transitory shock. Note also that this timing assumption implies that our analytical approach is akin to the common one of analyzing the response to shocks in deviations from a nonstochastic steady state.

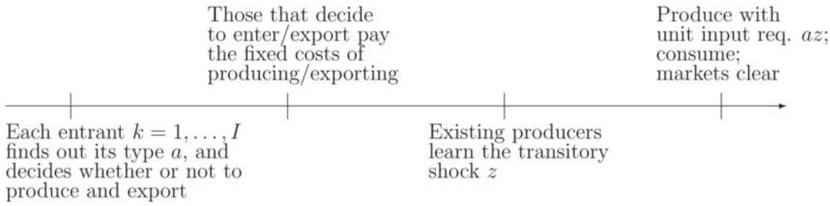

FIG. 4.—The timing of the economy

Because of fixed costs of serving a market, there is a cutoff unit input requirement a_{ij} above which firms in country j do not serve market i, defined as the level of $a(k) = a_{ij}$ such that the expected profits in equation (3) equal zero. To go forward with the analysis, we make the following two assumptions.

ASSUMPTION 1. The marginal firm is small enough that it ignores the impact of its own realization of $z(k)$ on the total expenditure X_i and the price level P_i in all potential destination markets $i = 1, \ldots, \mathcal{C}$.

ASSUMPTION 2. The marginal firm treats X_i and P_i as fixed (nonstochastic).

The first assumption is not controversial and has been made in the literature since Dixit and Stiglitz (1977) and Krugman (1980). The second assumption allows us to take X_i and P_i outside of the expectation operator. It amounts to assuming that the marginal entrepreneur ignores the volatility of aggregate output and the price level when deciding to enter a market.[10] Under these two assumptions, when we set (3) to zero and take the expectation over z, the zero-profit cutoff condition for serving market i from country j reduces to

[10] It is important to emphasize that these are assumptions placed on the behavior of the marginal entrepreneur. They allow us to compute the cutoffs for production and exporting a_{ij} as if the model were nonstochastic. This delivers substantial analytical and computational simplicity without affecting any of the main conclusions since in our model the economy is dominated by very large firms, and thus the marginal ones are not important for the aggregate outcomes. On the other hand, one may question our assumption about the behavior of the largest firms, namely, that markups are a constant multiple of marginal cost. If the largest firms in the economy are so large that their pricing decisions can affect the price level, their profit-maximizing prices will depart from the simple Dixit-Stiglitz constant markup benchmark. Note that, qualitatively, this critique applies to all implementations of the Dixit-Stiglitz framework and their extensions to heterogeneous firms. It is ultimately a quantitative question how much this force matters (Dixit and Stiglitz 1993; Yang and Heijdra 1993). While the full solution of our model under flexible markups would be impractical, and to our knowledge has not yet been implemented in this type of large-scale setting, we can perform a simple simulation that assesses the quantitative importance of allowing for variable markups in this setting. The Supplementary Web Appendix describes the exercise in detail and shows that, quantitatively, the deviations of flexible-markup prices from the constant-markup benchmark are very small even for the largest firms in small countries.

$$a_{ij} = \frac{\varepsilon - 1}{\varepsilon} \frac{P_i}{\tau_{ij} c_j} \left(\frac{X_i}{\varepsilon c_j f_{ij}} \right)^{1/(\varepsilon - 1)} \tag{4}$$

after normalizing the transitory shocks z such that $\mathrm{E}_z(z^{1-\varepsilon}) = 1$.

The equilibrium number of potential entrepreneurs $\bar{I}_j$ is pinned down by the familiar free-entry condition in each country. Entrepreneurs will enter until the expected profit equals the cost of finding out one's type:

$$\mathrm{E}\left[\sum_{i=1}^{\mathcal{C}} \mathbf{1}[a(k) \leq a_{ij}] \left\{ \frac{X_i}{\varepsilon P_i^{1-\varepsilon}} \left[\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} c_j a(k) z(k) \right]^{1-\varepsilon} - c_j f_{ij} \right\} \right] = c_j f_e \tag{5}$$

for each country j, where $\mathbf{1}[\cdot]$ is the indicator function.

Closing the model involves finding expressions for a_{ij}, P_i, w_i, and $\bar{I}_i$ for all i, $j = 1, \ldots, \mathcal{C}$. As an approximation, we solve for the equilibrium production allocation and price levels ignoring firm-specific transitory shocks. When we take the expectations over $a(k)$ and $z(k)$ and use the fact that $\mathrm{E}_z(z^{1-\varepsilon}) = 1$, the price levels become[11]

$$P_i = \left[\sum_{j=1}^{\mathcal{C}} \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{ij} c_j \right)^{1-\varepsilon} \bar{I}_j \Pr(a < a_{ij}) \mathrm{E}(a^{1-\varepsilon} | a < a_{ij}) \right]^{1/(1-\varepsilon)}. \tag{6}$$

We make the standard distributional assumption on productivity.

Assumption 3. Firm productivity $1/a$ follows a Pareto(b, θ) distribution: $\Pr(1/a < y) = 1 - (b/y)^{\theta}$, where b is the minimum value productivity can take, and θ regulates dispersion.

When we use the distributional assumption to compute the cumulative density functions (cdfs) and conditional expectations over a and plug in the expressions for a_{ij} in (4), the price levels become

$$\begin{aligned} P_i = {} & \frac{1}{b} \left[\frac{\theta}{\theta - (\varepsilon - 1)} \right]^{-1/\theta} \frac{\varepsilon}{\varepsilon - 1} \left(\frac{X_i}{\varepsilon} \right)^{-[\theta - (\varepsilon - 1)]/[\theta(\varepsilon - 1)]} \\ & \times \left[\sum_{j=1}^{\mathcal{C}} \bar{I}_j \left(\frac{1}{\tau_{ij} c_j} \right)^{\theta} \left(\frac{1}{c_j f_{ij}} \right)^{[\theta - (\varepsilon - 1)]/(\varepsilon - 1)} \right]^{-1/\theta}. \end{aligned} \tag{7}$$

The model is closed by assuming balanced trade in each country, which delivers a system of equations defining the vector of equilibrium wages w_i. The definition of equilibrium and the set of equilibrium conditions

[11] When the expressions for the price levels in (2) and (6) are compared, the set of varieties J_i available in country i comprise varieties coming from all countries serving market i. Thus, the expected number of varieties available in country i is equal to $\sum_{j=1}^{\mathcal{C}} \bar{I}_j \Pr(a < a_{ij})$.

for the complete two-sector model are laid out in Appendix B. In the remainder of this section, we use the relationships implied by the simple model above to illustrate the main mechanisms behind our results.

B. *Power Law in Firm Size and Aggregate Volatility in the Model and the Data*

Total sales in the economy is defined by

$$X = \sum_{k=1}^{I} x(a(k), z(k)), \tag{8}$$

where I is the total number of operating firms, $x(a(k), z(k))$ is the sales of firm k, and we omit the country subscripts. Appendix C shows that the standard deviation of the growth rate of aggregate sales, or more precisely of the deviation from the expected aggregate sales, is equal to

$$\mathrm{SD}_z\left[\frac{\Delta X}{\mathrm{E}_z(X)}\right] = \sigma\sqrt{h}, \tag{9}$$

where $h = \sum_{k=1}^{I} h(k)^2$ is the Herfindahl index of production shares of firms in this economy, and σ is the standard deviation of the growth rate of sales of an individual firm. This is the familiar expression for the standard deviation of a weighted sum of random variables and is the same as the one used by Gabaix (2011).[12]

This economy is granular, that is, idiosyncratic shocks to firms result in aggregate fluctuations, if the distribution of firm size follows a power law with an exponent sufficiently close to one in absolute value. In other words, firm sales x in the economy must conform to

$$\Pr(x > q) = \delta q^{-\zeta}, \tag{10}$$

where ζ is close to one. Gabaix (2011, proposition 2) shows that when the firm size distribution follows a power law with an exponent $-\zeta$, the economy is populated by $\mathcal{N}$ firms, and each firm has a standard deviation of sales growth equal to σ, the aggregate volatility given by (9) is proportional to $\sigma/\mathcal{N}^{1-1/\zeta}$ for $1 < \zeta < 2$ and to $\sigma/\log\mathcal{N}$ when $\zeta = 1$. This result means that when $\zeta < 2$ and thus the distribution of firm size has infinite variance, the conventional law of large numbers does not apply, and aggregate volatility decays in the number of firms $\mathcal{N}$ only very slowly. In other words, under finite variance in the firm size distribution, aggregate volatility decays at rate $\sqrt{\mathcal{N}}$ in the number of firms. But under Zipf's law—defined as $\zeta \approx 1$—it decays only at rate $\log\mathcal{N}$.

[12] Note that there are no aggregate shocks in the model, only the firm-specific idiosyncratic shocks.

In this paper, we take this statistical result for granted. This section relates it to our theoretical framework by first demonstrating how the parameters of the model can be calibrated to the observed distribution of firm size. Then we discuss the two key comparative statics: the role of country size and the role of trade openness in aggregate volatility.

It turns out that the baseline Melitz-Pareto model delivers a power law in firm size. We demonstrate the power law in an autarkic economy and then discuss how the distribution of firm size is affected by international trade. In our model, the expected sales of a firm as a function of its unit input requirement are $x(a) = Da^{1-\varepsilon}$, where the constant D reflects the size of domestic demand. Under the assumption that $1/a \sim \text{Pareto}(b, \theta)$, the power law follows

$$\begin{aligned}\Pr(x > q) &= \Pr(Da^{1-\varepsilon} > q)\\ &= \Pr\left(\frac{1}{a} > \left(\frac{q}{D}\right)^{1/(\varepsilon-1)}\right)\\ &= \left(\frac{b^{\varepsilon-1}D}{q}\right)^{\theta/(\varepsilon-1)},\end{aligned}$$

satisfying (10) for $\delta = (b^{\varepsilon-1}D)^{\theta/(\varepsilon-1)}$ and $\zeta = \theta/(\varepsilon - 1)$. This relationship is depicted in figure 5. Thus, our model economy will be granular if $\theta/(\varepsilon - 1)$ is close enough to one—the power law exponent in the data (see, e.g., Axtell 2001; di Giovanni, Levchenko, and Rancière 2011; di Giovanni and Levchenko 2013).

Gabaix (2011) shows that although aggregate volatility decays in the number of firms much more slowly than under the conventional law of large numbers, countries with a greater number of firms $\mathcal{N}$ will nonetheless have lower aggregate volatility. This forms the basis of the relationship between country size and aggregate volatility. Larger countries—those with higher L in our model—will feature a larger number of firms in equilibrium. Thus, they can be expected to have lower aggregate volatility. This can be demonstrated most transparently in the autarky equilibrium. When the number of countries is set to $\mathcal{C} = 1$ and the Pareto distributional assumption is used, equation (5) can be used to compute the production cutoff a_{aut} consistent with free entry. With zero net aggregate profits, the total expenditure in the economy is equal to $X = L/\beta$, where we set the wage to be the numeraire. Equations (4) and (7) then imply that the equilibrium number of entrants $\bar{I}_{\text{aut}}$ is proportional to

$$\bar{I}_{\text{aut}} \sim L^{1/\{1-[(1-\beta)/\beta][1/(\varepsilon-1)]\}}. \tag{11}$$

This is the well-known result that the number of firms increases in country size, measured by L. It is immediate that without input-output

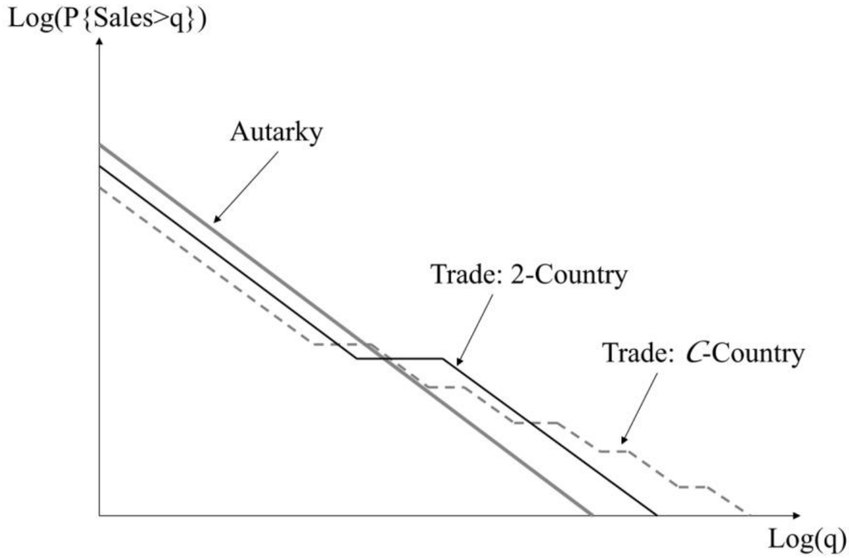

FIG. 5.—The analytical power law in the Melitz-Pareto model. This figure depicts the distribution of firm size, measured by sales, and how it changes as it moves from autarky to a two-country trade equilibrium and finally to a $\mathcal{C}$-country trade equilibrium. In the two-country case, there is a single productivity cutoff, above which firms export abroad. Compared to autarky, there is a higher probability of finding larger firms above this cutoff. In the $\mathcal{C}$-country case, with multiple export markets there will be cutoffs for each market, with progressively more productive firms exporting to more and more markets and growing larger and larger relative to domestic GDP.

linkages ($\beta = 1$), the relationship is simply proportional.[13] The presence of input-output linkages actually tends to raise this elasticity above one: as long as $\beta\varepsilon > 1$, the number of firms responds more than proportionately to the increase in market size. The condition that $\beta\varepsilon > 1$ is akin to the "no black hole" assumption (Fujita, Krugman, and Venables 2001). Otherwise, if $\beta\varepsilon < 1$, increasing returns are so strong that countries with higher L actually have a smaller number of entrants.[14] We impose the restriction that $\beta\varepsilon > 1$ throughout. It is likely to be comfortably satisfied in the data, as available estimates put β in the range of 0.5, while ε is typically assumed to be around six (see Sec. IV.A for details).[15] The

[13] In that case, the solution for the equilibrium number of entrants has the particularly simple form

$$\bar{I}_{\text{aut}} = \frac{L}{\varepsilon f_e}\frac{\varepsilon - 1}{\theta}.$$

[14] The model does not have a solution when $\beta\varepsilon = 1$, and the equilibrium $\bar{I}_{\text{aut}}$ is discontinuous in $\beta\varepsilon$ in the neighborhood of $\beta\varepsilon = 1$: as $\beta\varepsilon \to 1$ from below, $\bar{I}_{\text{aut}}$ goes to zero; but as $\beta\varepsilon \to 1$ from above, $\bar{I}_{\text{aut}}$ goes to infinity.

[15] One may wonder whether the larger number of entrants $\bar{I}$ actually translates into a larger number of operating firms, since not all entrants decide to produce. The number of

equilibrium relationship (11) combined with Zipf's law in firm size thus forms the basis for the first main result of the paper: smaller countries will have fewer firms and, thus, higher aggregate volatility.

C. *International Trade and Aggregate Volatility*

How does international trade affect the distribution of firm size and therefore aggregate volatility? As first demonstrated by Melitz (2003), the distribution of firm size becomes more unequal under trade: compared to autarky, the least productive firms exit, and only the most productive firms export abroad. Owing to competition from foreign varieties, domestic sales and profits decrease. Thus, as a country opens to trade, sales of most firms shrink, while the largest firms grow larger as a result of exporting.[16] Figure 5 depicts this effect. In the two-country case, there is a single productivity cutoff, above which firms export abroad. Compared to autarky, there is a higher probability of finding larger firms above this cutoff. In the $\mathcal{C}$-country case with multiple export markets, there will be cutoffs for each market, with progressively more productive firms exporting to more and more markets and growing larger and larger relative to domestic GDP. Thus, if the distribution of firm sales follows a power law and the economy is granular, international trade has the potential to increase the size of the largest firms, in effect creating a "hypergranular" economy, with clear implications for the relationship between trade openness and aggregate volatility. All else equal, this "selection into exporting" effect implies that after trade opening, aggregate volatility increases.

operating firms is given by $\bar{I}_{\text{aut}} G(a_{\text{aut}})$, where $G(\cdot)$ is the cdf of a. The solution to a_{aut} does not depend on L in this model, and thus the number of actual operating firms is proportional to $\bar{I}_{\text{aut}}$.

[16] Firm-level studies of dynamic adjustment to trade liberalization appear to find empirical support for these predictions. Pavcnik (2002) provides evidence that trade liberalization led to a shift in resources from the least to the most productive firms in Chile. Bernard, Jensen, and Schott (2003) show that a fall in trade costs leads to both exit by the least productive firms and entry by firms into export markets. In addition, existing exporters ship more abroad. A recent contribution by Holmes and Stevens (2010) shows that in the United States, in some sectors the large firms are the ones suffering the most from foreign competition, because smaller firms are highly specialized boutique operations that are less affected by imports than the large factories producing standardized products with close foreign substitutes. The point made by Holmes and Stevens is a very important one, but it can be thought of as one about industrial classification: large factories and boutique ones produce different types of goods, which face very different market structures—competitive environments, trade costs, and so on. This comes through most clearly in the modeling approach adopted in that paper, in which it classifies the small boutique producers as nontradable. Thus, the Holmes and Stevens finding can be easily reconciled with our complete two-sector model, described in App. B and used in the quantitative analysis, by assuming that the standardized producers are part of the tradable sector, while the boutique producers are part of the nontradable sector. Indeed, this is very close to the assumption that Holmes and Stevens actually adopt in their model.

Before moving on to the quantitative assessment of the relationships between country size, international trade, and volatility illustrated above, we allude to another mechanism through which trade can affect volatility in a model with free entry. When a country opens to trade, the possibility of getting a sufficiently high productivity draw and becoming an exporter induces more potential entrepreneurs to enter and draw their productivity: $\bar{I}$ rises. Because aggregate volatility decreases in the number of firms, this "net entry" effect will tend to decrease volatility when a country opens to trade. As Section V demonstrates, however, this effect is quite small quantitatively: the impact of international trade on aggregate volatility is virtually the same whether we allow new net entry after opening or not.[17]

IV. Quantitative Assessment

Though the analytical results obtained in a one-sector model are informative, we would like to evaluate quantitatively the importance of these mechanisms and exploit the rich heterogeneity among the countries in the world. In order to do this, we numerically implement a multicountry model that extends the framework in Section III to include a nontraded sector with intermediate input linkages both within and between sectors. Since only the minority of economic activity takes place in sectors with substantial cross-border trade, including an explicitly nontraded sector in the quantitative exercise is especially important for evaluating the impact of international trade on volatility.

In particular, suppose that in each country there are two broad sectors, the tradable T and the nontradable N. Consumer preferences are Cobb-Douglas in CES aggregates of N and T, with the share of N in

[17] We can use a back-of-the-envelope calculation to illustrate why quantitatively free entry plays such a minor role for the impact of trade. When countries are symmetric ($L_i = L$, $f_{ii} = f$ for all i, and $\tau_{ij} = \tau$, $f_{ij} = f^X$ for all i, j), the number of entrants under trade is

$$\bar{I}_{\text{trade}} = [1 + (\mathcal{C} - 1)\tau^{-\theta}(f/f^X)^{[\theta-(\varepsilon-1)]/(\varepsilon-1)}]^{[(1-\beta)/\beta\theta](1/\{1-[(1-\beta)/\beta][1/(\varepsilon-1)]\})}\bar{I}_{\text{aut}}.$$

Trade opening thus increases the number of entrants relative to autarky under the maintained assumption that $\beta\varepsilon > 1$, since the term in brackets is larger than one. At reasonable parameter values, the exponent on the bracket in the equation above is quite small, and thus the change in $\bar{I}$ from autarky to trade is modest. Furthermore, even if the number of operating firms increases by as much as the difference between $\bar{I}_{\text{aut}}$ and $\bar{I}_{\text{trade}}$, this effect alone would amount to a proportional reduction in volatility of only $1 - (\bar{I}_{\text{trade}}/\bar{I}_{\text{aut}})^{1-1/\zeta}$, which is very small if ζ is close to one. To get a sense of the magnitudes, we plug in the parameter values for τ, f, f_X, θ, ε, and β from the quantitative model below (see Sec. IV.A and table 1). It turns out that with 100 symmetric countries (each thus accounting for 1 percent of world GDP), $\bar{I}_{\text{trade}}/\bar{I}_{\text{aut}} = 1.21$, which alone would imply a reduction in aggregate volatility of only 0.9 percent compared to autarky. With 200 countries (each 0.5 percent of world GDP), the reduction is 1.4 percent. These are upper bounds because the number of actual operating firms will increase by less than the term in brackets above, as the production cutoffs will also become more stringent under trade.

final expenditure equal to α. Intermediate inputs are also Cobb-Douglas in the N and T aggregates, with the share of N equal to η_s in sector $s = N, T$. The share of labor in total spending on inputs, β_s, will also now vary by sector. The rest of the model remains unchanged. Appendix B presents the complete description of the equations defining the equilibrium in the two-sector model.

A. *Calibration*

We numerically implement the economy under the following parameter values (see table 1 for a summary). The elasticity of substitution is $\varepsilon_s = 6$. Anderson and van Wincoop (2004) report available estimates of this elasticity to be in the range of 3–10, and we pick a value close to the middle of the range. The key parameter is θ_s, as it governs the slope of the power law. As described above, in this model firm sales follow a power law with the exponent equal to $\theta_s/(\varepsilon_s - 1)$. In the data, firm sales follow a power law with the exponent close to one. Axtell (2001) reports the value of 1.06, which we use to find θ_s given our preferred value of ε_s: $\theta_s = 1.06 \times (\varepsilon_s - 1) = 5.3$. We set both the elasticity of substitution and the Pareto exponent to be the same in the N and the T sectors. Appendix B justifies in detail the calibration of the two-sector model parameters to the observed firm size distributions.

We set the share of nontradables in consumption $\alpha = 0.65$. This is the mean value of services value added in total value added in the database

TABLE 1
Parameter Values for the Calibrated Model

Parameter	Baseline	Source
ε	6	Anderson and van Wincoop (2004)
θ	5.3	Axtell (2001): $\theta/(\varepsilon - 1) = 1.06$
α	.65	Yi and Zhang (2010)
$\{\beta_N, \beta_T\}$	$\{.65, .35\}$	1997 US Benchmark Input-Output Table
$\{\eta_N, \eta_T\}$	$\{.77, .35\}$	1997 US Benchmark Input-Output Table
τ_{ij}	2.30	Helpman et al. (2008)
f_{ii}	14.24	World Bank Doing Business Indicators: normalizing $f_{\mathrm{US,US}}$ so that nearly all firms in the United States produce
f_{ij}	7.20	World Bank Doing Business Indicators: normalizing $f_{\mathrm{US,US}}$ so that nearly all firms in the United States produce
f	34.0	To match 7 million firms in the United States (US Economic Census)
σ	.1	Standard deviation of sales growth of the top 100 firms in Compustat

Note.—For ε, robustness checks include $\varepsilon = 4$ and $\varepsilon = 8$. For θ, robustness checks include $\theta/(\varepsilon - 1) = 1.5$ and $\varepsilon = 6$, so that $\theta = 6.5$. For τ_{ij}, f_{ii}, and f_{ij} the table reports the average in our sample of 50 countries. For τ_{ij}, $\tau_{ij} = \tau_{ji}$. Trade costs are adjusted by a constant ratio to match the median level of openness across the 50-country sample. Robustness checks include σ varying with firm sales: $\sigma = Ax^{-\xi}$, where $\xi = 1/6$.

compiled by the Groningen Growth and Development Center and extended to additional countries by Yi and Zhang (2010). It is the value also adopted by Alvarez and Lucas (2007). The values of β_N and β_T—share of labor/value added in total output—are calibrated using the 1997 US Benchmark Input-Output Table. We take the Detailed Make and Use tables, featuring more than 400 distinct sectors, and aggregate them into a two-sector Direct Requirements Table. This table gives the amount of N, T, and factor inputs required to produce a unit of final output. Thus, β_s is equal to the share of total sector s output that is not used to pay for intermediate inputs, that is, the payments to factors of production. According to the US Input-Output Matrix, $\beta_N = 0.65$ and $\beta_T = 0.35$: the traded sector is considerably more input-intensive than the nontraded sector. The shares of nontraded and traded inputs in both sectors are also calibrated from the US Input-Output Table. According to the data, more than 75 percent of the inputs used in the N sector come from the N sector itself ($\eta_N = 0.77$), while only 35 percent of T-sector inputs are nontradable ($\eta_T = 0.35$). Nonetheless, these values still leave substantial room for cross-sectoral input-output linkages.

To calibrate the values of τ_{ij} for each pair of countries, we use the gravity estimates from the empirical model of Helpman, Melitz, and Rubinstein (2008). To take a stand on the values of f_{ii}^s and f_{ij}^s, we follow di Giovanni and Levchenko (2013) and use the information on entry costs from the World Bank's Doing Business Indicators database. The data sources and the details of the calibration of τ_{ij} and f_{ij}^s are described in Appendix A.

Finally, we set the value of the "exploration cost" f_e such that the equilibrium number of operating firms in the United States is about 7 million. According to the 2002 US Economic Census, there were 6,773,632 establishments with a payroll in the United States. There are an additional 17,646,062 business entities that are not employers, but they account for less than 3.5 percent of total shipments. Thus, while the United States may have many more legal entities than what we assume here, 7 million is a number sufficiently high as to let us consider consequences of granularity. Since we do not have information on the total number of firms in other countries, we choose to set f_e to be the same in all countries. In the absence of data, this is the most agnostic approach we could take. In addition, since f_e represents the cost of finding out one's abilities, we do not expect it to be affected by policies and thus differ across countries. The resulting value of f_e is 15 times higher than $f_{\mathrm{US,US}}^s$ and 2.4 times higher than the average f_{ii}^s in the rest of the sample. The finding that the ex ante fixed cost of learning one's type is much higher than the ex post fixed cost of production is common in the quantitative models of this type (see, e.g., Ghironi and Melitz 2005).

We carry out the analysis on the sample of the largest 49 countries by total GDP plus the fiftieth that represents the rest of the world. These 49 countries together cover 97 percent of world GDP. We exclude the entrepôt economies of Hong Kong and Singapore, both of which have total trade well in excess of their GDP because of significant reexporting activity. Thus, our model is not intended to fit these countries. (We do place them into the rest-of-the-world category.) The country sample, sorted by total GDP, is reported in table 2.

B. *Model Solution and Simulation Method*

In order to solve the model numerically, we must find the wages and price indices for each country, w_i, P_i^N, and P_i^T, that satisfy equations (B1), (B2), and (B3) jointly with the values of $\bar{I}_i^N$ and $\bar{I}_i^T$ that satisfy equations (5) for each sector. The system is nonreducible such that all of the prices and numbers of entrants must be solved simultaneously. Note that in this step the equilibrium values are computed under the assumption that the

TABLE 2
Top 49 Countries and the Rest of the World in Terms of Total GDP

Country	GDP/World GDP	Country	GDP/World GDP
United States	.300	Indonesia	.006
Japan	.124	South Africa	.006
Germany	.076	Norway	.006
France	.054	Poland	.005
United Kingdom	.044	Finland	.005
Italy	.041	Greece	.004
China	.028	Venezuela, RB	.004
Canada	.026	Thailand	.004
Brazil	.021	Portugal	.003
Spain	.020	Colombia	.003
India	.017	Nigeria	.003
Australia	.016	Algeria	.003
Russian Federation	.015	Israel	.003
Mexico	.015	Philippines	.003
Netherlands	.015	Malaysia	.002
Korea, Republic	.011	Ireland	.002
Sweden	.010	Egypt, Arab Republic	.002
Switzerland	.010	Pakistan	.002
Belgium	.009	Chile	.002
Argentina	.008	New Zealand	.002
Saudi Arabia	.007	Czech Republic	.002
Austria	.007	United Arab Emirates	.002
Iran, Islamic Republic	.007	Hungary	.002
Turkey	.007	Romania	.002
Denmark	.006	Rest of the world	.027

Note.—Ranking of top 49 countries and the rest of the world in terms of total US$ GDP, with the average share in world GDP over 1970–2006. Source: World Bank World Development Indicators.

model aggregates take on their expected values. That is, this step ignores any variation in P_i^s's, $\bar{I}_i^s$'s, and a_{ij}^s's that would arise from one random draw of a vector of a's to another—a common approach in monopolistic competition models.

Using the equilibrium equations and the chosen parameter values, we can solve the full model for a given vector of L_i. For finding the values of L_i, we follow the approach of Alvarez and Lucas (2007). First, we would like to think of L_i not as population per se but as "equipped labor," to take explicit account of total factor productivity (TFP) and capital endowment differences between countries. To obtain the values of L_i that are internally consistent in the model, we start with an initial guess for L_i for all $i = 1, \ldots, \mathcal{C}$ and use it to solve the full model. Given the solution for wages, we update our guess for L_i for each country in order to match the GDP ratio between each country i and the United States. Using the resulting values of L_i, we solve the model again to obtain the new set of wages and iterate to convergence (Alvarez and Lucas 2007). Thus, our procedure generates vectors w_i and L_i in such a way as to match exactly the relative total GDPs of the countries in the sample. In practice, the results are close to simply equating L_i to the relative GDPs. In this procedure, we must normalize the population of one of the countries. We thus set L_{US} to its actual value of 291 million as of 2003 and compute L_i of every other country relative to this US value. An important consequence of this approach is that countries with higher TFP and capital abundance will tend to have a greater number of potential productivity draws $\bar{I}_i^s$, all else equal, since our procedure will effectively give them a higher L_i. This is akin to the assumption adopted by Alvarez and Lucas (2007) and Chaney (2008) that the number of productivity draws is a constant multiple of equipped labor L_i. The difference in our approach is that though we take labor-cum-productivity to be the measure of market size, we solve for $\bar{I}_i^N$ and $\bar{I}_i^T$ endogenously within the model.

Having solved the model given the data on country GDPs and trade costs, we now simulate it using random productivity draws for each firm in each economy. Namely, in each country i and sector s we draw $\bar{I}_i^s$ productivities from a Pareto(b_s, θ_s) distribution. For each firm, we use the cutoffs a_{ji}^s for serving each market j (including its own market $j = i$) given by equation (4) to determine whether the firm operates and which, if any, foreign markets it serves. We next calculate the total sales of each firm as the sum of its sales in each market it serves and compute the Herfindahl index of firm sales in country i. Since the distribution of firm productivities gives rise to a highly skewed distribution of firm sales, there is variation in the Herfindahl index from simulation to simulation, even though we draw as many as 7 million operating firms in a given country; note that this number is the total for the N and T sectors, where we take independent draws for each sector. We thus repeat the exercise 1,001 times and take the median values of the Herfindahl index in each country. In parallel, we also compute the Herfindahl index of firm sales in autarky for each country, which will allow us to gauge the contribution of international trade to aggregate volatility. Given these values of the Herfindahl index h, we can then construct each country's aggregate volatility under trade and in autarky using the formula for the standard deviation of aggregate output growth (9) and a realistic value of σ. Following Gabaix (2011), we set $\sigma = 0.1$, although since in this paper we will not exploit any variation in σ across countries, none of the results will be driven by this choice.

It is worth making a comparison between our procedure and the one adopted by Eaton et al. (2012). The authors simulate the firm behavior first, starting with the lowest cost and working their way up. That procedure makes it easier to then calculate entry cutoffs and price indices for each country conditional on the firm-level draws. The advantage of this approach is that in calculating the price index, the issue of an exploding integral does not arise even if $\theta_s < \varepsilon_s - 1$. A disadvantage is that, to keep the procedure tractable, Eaton et al. need to impose additional assumptions so that the total spending in each country and the wage can be taken as exogenous. This would be problematic in our setting since for us the equilibrium number of entrants is an endogenous, and central, outcome driving the results on the cross-country variation in volatility.

C. Model Fit

We assess the model fit along three dimensions: (i) overall and bilateral trade volumes, (ii) the relationship between country size and the size of the largest firms in each country, and (iii) the share of exporting firms in the economy.

Figure 6*A* reports the scatter plot of bilateral trade ratios, $\pi_{ij} = X_{ij}/w_i L_i$. Note that since in the data we have bilateral trade only as a share of GDP, not of total sales, we compute the same object in the model. This captures both the distinction between trade, which is recorded as total value, and GDP, which is recorded as value added, as well as the fact that there is a large nontraded sector in both the model and the data. On the horizontal axis is the natural logarithm of π_{ij} that comes from the model, while on the vertical axis is the corresponding value of that bilateral trade flow in the data. Hollow dots represent exports from one country to another, π_{ij}, $i \neq j$. Solid dots, at the top of the scatter plot, represent sales of domestic firms as a share of domestic absorption, π_{ii}. It is especially important that we reproduce the variation in the overall trade openness $(1 - \pi_{ii})$ since that will drive the contribution of trade to the aggregate volatility in each country. Figure 6*B* plots the actual values of $1 - \pi_{ii}$ against those implied by the model. For convenience, we add a 45-degree line to both plots.

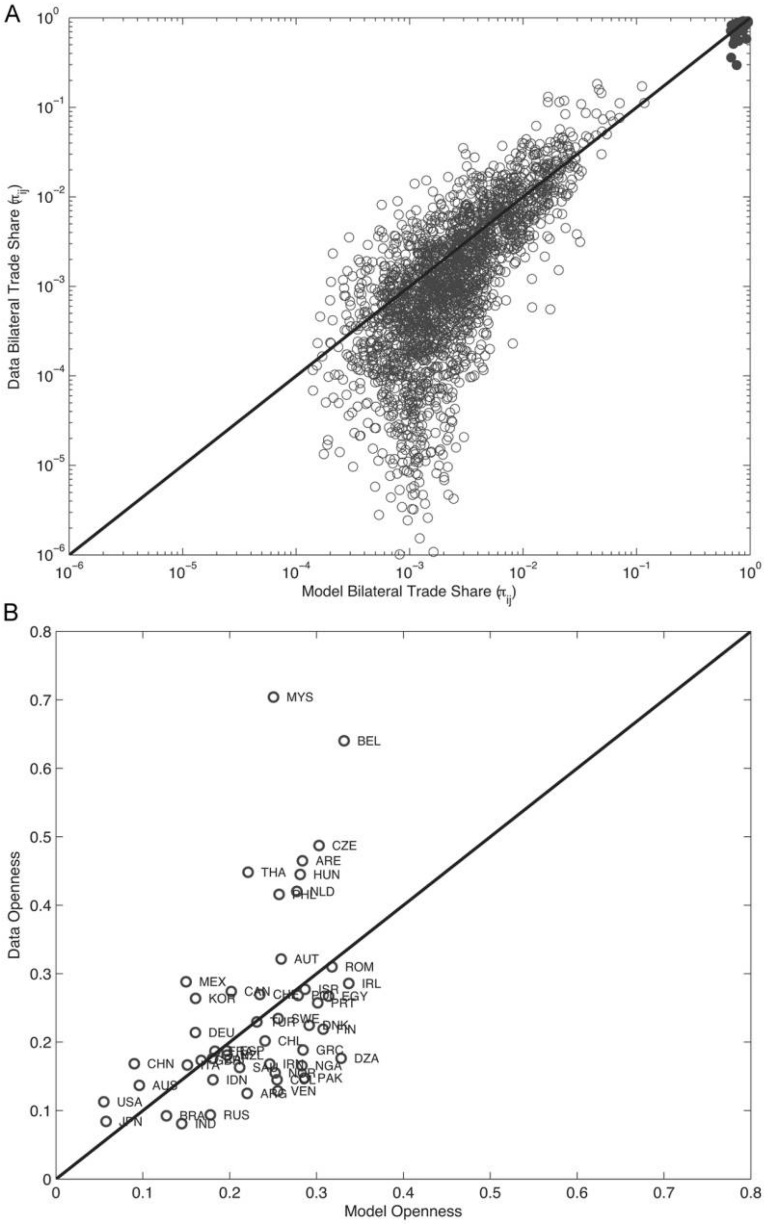

FIG. 6.—Bilateral trade shares and trade openness: data and model predictions. Panel *A* reports the scatter plot of domestic output (π_{ii}) and bilateral trade (π_{ij}), both as a share of country *i* GDP. Solid dots represent observations of π_{ii}, and hollow dots represent bilateral trade observations (π_{ij}). Both axes are in log scale. Panel *B* reports the scatter plot of total imports as a share of GDP. In both panels, the values implied by the model are on the horizontal axis; actual values are on the vertical axis, and the line through the data is the 45-degree line. Source: International Monetary Fund's Direction of Trade Statistics.

Table 3 compares the means and medians of π_{ii} and π_{ij}'s for the model and the data and reports the correlations between the two. The correlation between domestic shares π_{ii} calculated from the model and those in the data for this sample of countries is around .48. The correlation between export shares, π_{ij}, is actually higher at .78.[18] Since we use estimated gravity coefficients together with the actual data on bilateral country characteristics to compute trade costs, it is not surprising that bilateral trade flows implied by our model are closely correlated to those in the data given the success of the empirical gravity relationship. Nonetheless, since the gravity estimates we use come from outside of our calibration procedure, it is important to check that our model delivers outcomes similar to observed trade volumes.

We next assess whether the model reproduces the relationships between country size and the relevant features of the firm size distributions demonstrated in figure 3. In the data, log country size is negatively and significantly related to the log Herfindahl index of firm sales and positively and significantly related to the size of the 10 largest firms and the size of the largest firm in the economy. We can compare the data to the same relationships inside our model. It turns out that in the model the elasticity of the Herfindahl index with respect to country size is -0.135, which is right in between the Herfindahl–country size elasticity of -0.114 (sample of countries with more than 1,000 firms) and -0.284 (sample of countries with more than 100 firms) in figure 3*A*. Turning to the size of the largest firms, our model produces an elasticity of the 10 largest firms to country size of 0.903 and of the single-largest firm to country size of 0.908. These are close to the range of elasticities produced by the data: 0.888–1.006 for the 10 largest firms (fig. 3*B*) and 0.838–0.906 (fig. 3*C*) for the single-largest firm.

Finally, we use the model solution to calculate the percentage of firms that export in the total economy as well as in the tradable sector. In particular, the total number of exporters in country i equals $\bar{I}_i^T \times (b_T \max_{j \neq i}\{a_{ji}^T\})^{\theta_T}$. The total number of firms operating in the tradable sector equals $\bar{I}_i^T \times (b_T \max_j\{a_{ji}^T\})^{\theta_T}$ and in the nontradable sector $\bar{I}_i^N \times (b_N a_{ii}^N)^{\theta_N}$. We would like to compare the export participation shares in the model to what is found in the data. Unfortunately, there is no systematic empirical evidence on these shares across countries (and time). However, we have examined available data and existing literature and found these shares for eight countries: United States, Germany, France, Argentina, Colombia, Ireland, Chile, and New Zealand. Table 4 compares

[18] We also experimented with increasing the number of countries in the simulation to 60. The model fits the data well, but there are more zeros in bilateral trade data in the 60-country sample compared to the 50-country one. (With 50 countries, among the 2,500 possible unidirectional bilateral trade flows, only 18 are zeros.) Since our model does not generate zero bilateral trade outcomes, we stick with the largest 49 countries in our analysis.

TABLE 3
BILATERAL TRADE SHARES: DATA AND MODEL PREDICTIONS

	Model		Data
Domestic sales as a share of GDP (π_{ii}):			
Mean	.7900		.7520
Median	.7717		.7921
Corr(model, data)		.4783	
Imports as a share of GDP (π_{ij}):			
Mean	.0043		.0047
Median	.0021		.0047
Corr(model, data)		.7799	

NOTE.—This table reports the means and medians of domestic output (top panel) and bilateral trade (bottom panel), both as a share of domestic GDP, in the model and in the data. Source: International Monetary Fund's Direction of Trade Statistics.

TABLE 4
EXPORT PARTICIPATION: DATA AND MODEL PREDICTIONS FOR THE WHOLE ECONOMY AND THE TRADABLE SECTOR

	MODEL		DATA	
COUNTRY	Total (1)	Tradable (2)	Total (3)	Tradable (4)
United States	.010	.018	.040	.150
Germany	.111	.238	.100	. . .
France	.029	.065	.040	.090
Argentina	.112	.352	. . .	.422
Colombia	.148	.548	. . .	.363
Ireland	.332	1.000	. . .	.740
Chile	.095	.335	.105	. . .
New Zealand	.062	.189	.051	.135

SOURCE.—For the United States, data are imputed on the basis of publicly available US Economic Census data on the numbers of firms by sector, together with the summary statistics for the numbers of exporters reported in Bernard et al. (2007). Data for France are based on authors' calculations using the French census data in di Giovanni et al. (2011). Data for Germany are from Arndt, Buch, and Mattes (2009, table A2). Data for Argentina come from Bustos (2011, table D.1). For New Zealand, data come from Fabling and Sanderson (2008, table 4). Data on Ireland come from Fitzgerald and Haller (2010, table 1). Data for Chile come from private communication with Miguel Fuentes at the Central Bank of Chile. Data for Colombia come from private communication with Jorge Tovar at the Universidad de los Andes.

NOTE.—This table compares, for selected countries, the share of exporters among all firms in the model (col. 1) and the share of exporters among the tradable sector firms in the model (col. 2) with available estimates of corresponding shares in existing literature. Since for some countries data are reported relative to all the firms in the economy while for other countries they are reported relative to all the firms in the traded sector, col. 3 (data) should be compared to col. 1 (model) and col. 4 (data) should be compared to col. 2 (model).

the export participation shares produced by the model to those found in the data in this subset of countries. Columns 1 and 2 report the values in the model, with the shares of exporters relative to all the firms in the economy in column 1 and in the tradable sector only in column 2. Data sources differ across countries; in particular, the shares of exporting firms are sometimes reported relative only to all firms in the economy (which we record in col. 3) and sometimes relative to all the firms in the tradable sector (which we record in col. 4). Thus, data in column 3 should be compared to model outcomes in column 1, while data in column 4 should be compared to model outcomes in column 2.

In both the data and the model, larger countries tend to have fewer exporters relative to the overall number of firms (compare the United States to Colombia); countries closer to large markets tend to have higher shares of exporters compared to faraway countries (compare Ireland to New Zealand). In most cases the model-implied value is close to the data. We should note that by making ad hoc adjustments to trade costs in individual countries, we can match each and every one of these numbers exactly. We do not do so because this information is not available systematically for every country in our sample and because the available firm-level data themselves are noisy. Instead, we take trade costs as implied by a basic gravity model and the variation in fixed costs as implied by the Doing Business Indicators, an approach that is rather straightforward and does not involve any manual second-guessing.

D. *Main Results: Country Size and Trade Openness*

As would be expected, the level of aggregate volatility in the model is lower than what is observed in the data, since in the model all volatility comes from idiosyncratic shocks to firms. Column 1 of table 5 reports the ratio of the aggregate volatility implied by the model to the actual GDP volatility found in the data. It ranges between 0.14 and 0.72, with a value of 0.377 for the United States, almost identical to what Gabaix (2011) finds using a very different methodology. Note that the variation in aggregate volatility in the model across countries is generated by differences in country size as well as variation in bilateral trade costs.

How well can the model reproduce the empirical relationship between aggregate volatility and country size? Figure 7 plots volatility as a function of country size in the data and the model. Note that since the level of aggregate volatility in the model does not match up with the level in the data, this graph is informative about the comparison of only slopes, not intercepts. In the data the elasticity of GDP volatility with respect to country size is -0.139 (σ_{GDP}) in this sample of countries. Appendix table D1 reports the results of estimating the volatility-size relationship in the data for various country samples and with and without controls. The

TABLE 5
International Trade and Aggregate Volatility

Country	Trade/ Actual (1)	Trade/ Autarky (2)	Country	Trade/ Actual (1)	Trade/ Autarky (2)
United States	.377	1.035	Indonesia	.376	1.060
Japan	.405	1.014	South Africa	.535	1.109
Germany	.582	1.080	Norway	.716	1.137
France	.559	1.098	Poland	.377	1.114
United Kingdom	.476	1.076	Finland	.437	1.109
Italy	.463	1.098	Greece	.414	1.116
China	.280	1.024	Venezuela, RB	.285	1.070
Canada	.446	1.077	Thailand	.337	1.099
Brazil	.311	1.045	Portugal	.379	1.068
Spain	.550	1.061	Colombia	.646	1.118
India	.371	1.064	Nigeria	.274	1.172
Australia	.513	1.051	Algeria	.271	1.156
Russian Federation	.144	1.099	Israel	.513	1.131
Mexico	.329	1.052	Philippines	.439	1.107
Netherlands	.693	1.104	Malaysia	.371	1.095
Korea, Republic	.296	1.059	Ireland	.457	1.087
Sweden	.634	1.099	Egypt, Arab Republic	.513	1.192
Switzerland	.548	1.107	Pakistan	.630	1.165
Belgium	.713	1.072	Chile	.262	1.119
Argentina	.219	1.091	New Zealand	.531	1.114
Saudi Arabia	.168	1.069	Czech Republic	.330	1.095
Austria	.716	1.066	United Arab Emirates	.178	1.089
Iran, Islamic Republic	.189	1.097	Hungary	.399	1.114
Turkey	.254	1.157	Romania	.242	1.218
Denmark	.612	1.156			

Note.—Trade/Actual reports the ratio of aggregate volatility implied by the model under trade to the actual volatility of per capita GDP growth. In calculating volatility in the model, this column assumes that the standard deviation of firm-level sales growth rate is equal to $\sigma = 0.1$. Trade/Autarky reports the ratio of volatility in the model under trade to the volatility under autarky for each country. Aggregate volatility referring to either the model or the data stands for the standard deviation of the aggregate growth rate.

baseline coefficient used in figure 7 comes from the 50-country sample and controlling for income per capita. Our calibrated model produces an elasticity of -0.135 (σ_T), which is extremely close to the one in the data though slightly below it in absolute terms.

We now assess the contribution of international trade to aggregate volatility in our sample of countries. Our model yields not only the aggregate volatility in the simulated trade equilibrium but also the aggregate volatility in autarky. As a preview of the results on the impact of trade openness, figure 7 reports the volatility-size relationship in autarky. Without trade this relationship is somewhat flatter: the elasticity of volatility with respect to country size in autarky is -0.115 (σ_A), lower than the -0.139 in the data. Thus, it appears that openness helps the model match the slope of the size-volatility relationship: without trade, smaller countries would be relatively less volatile than they actually are.

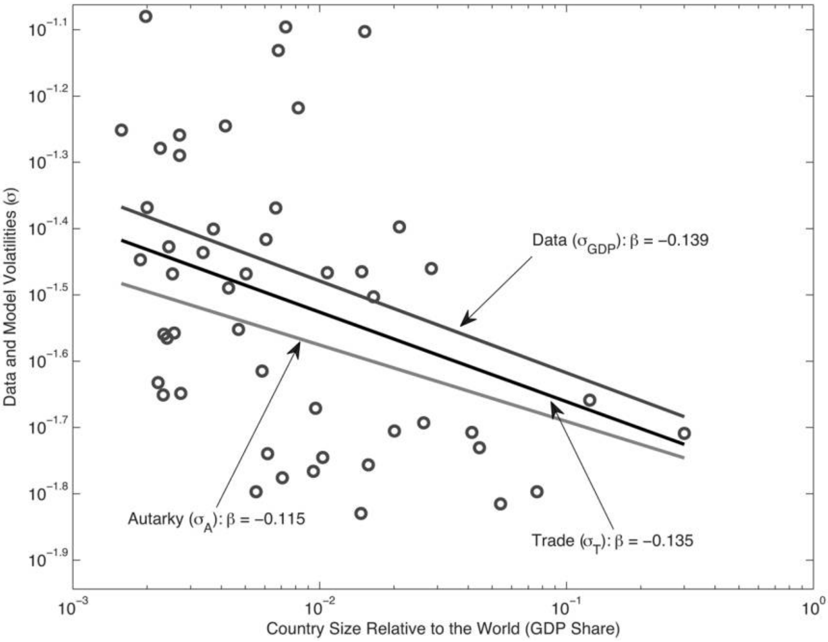

FIG. 7.—Volatility and country size: data and model predictions. This figure plots the relationship between country size and aggregate volatility implied by the data (conditioning on per capita GDP), the model under trade, and the model in autarky. The dots represent actual observations of volatility. Note that the data points and regression line are shifted by a constant for ease of visual comparability with the model regressions lines. Source: World Bank World Development Indicators.

Column 2 of table 5 reports the ratio of the volatility under the current trade regime to the volatility in autarky in each country in the sample. In the table, countries are ranked by size in descending order. We can see that international trade contributes very little to overall GDP volatility in the United States. The country is so large and trade volumes are so low (relative to total output) that its volatility under trade is only 1.035 times higher than it would be in a complete absence of trade. Similar results obtain for other very large economies, such as Japan and China. By contrast, smaller, centrally located countries experience substantially higher volatility compared to autarky. For instance, in a country such as Romania, the volatility under trade is some 22 percent higher than it would be in autarky, and in Turkey, Denmark, and Norway, it is 14–16 percent higher. In between are small but remote countries. South Africa, Argentina, and New Zealand experience aggregate volatility that is about 10 percent higher than it would have been in autarky.

Finally, we investigate how well the model predicts the actual GDP volatility found in the data. Table 6 presents regressions of actual vola-

TABLE 6
Aggregate Volatility: Data and Model Predictions

	Dependent Variable: Log(GDP Volatility)			
	(1)	(2)	(3)	(4)
Log(σ_T)	1.578**	1.365**	1.099**	.765**
	(.244)	(.321)	(.287)	(.274)
Log(GDP per capita)		−.093	−.098	−.146*
		(.073)	(.065)	(.060)
Log(risk content of exports)			.100$^+$	−.064
			(.053)	(.052)
Log(Herfindahl of production)				−.134
				(.217)
Constant	3.490**	3.417**	2.994**	.282
	(1.092)	(1.145)	(1.079)	(1.045)
Observations	49	49	47	35
R^2	.353	.378	.477	.450

Note.—The dependent variable is the standard deviation of per capita GDP growth over the period 1970–2006. The term σ_T is the aggregate volatility implied by the simulated model. GDP per capita is the PPP-adjusted per capita GDP. Risk content of exports is the measure of the volatility of a country's export pattern taken from di Giovanni and Levchenko (2012). Herfindahl of production is the Herfindahl index of industry-level production shares, taken from di Giovanni and Levchenko (2009). Robust standard errors are in parentheses.

$^+$ Significant at 10 percent.
* Significant at 5 percent.
** Significant at 1 percent.

tility of per capita GDP growth over the period 1970–2006 against the one predicted by the model (σ_T), with all variables in natural logs. Column 1 includes no controls. The relationship is positive and highly significant. The fit of this simple bivariate relationship is remarkably high ($R^2 = .353$), given that in the model variation in σ_T is driven only by country size, trade barriers, and fixed costs. The model uses no information on any type of aggregate shocks (TFP, monetary, or fiscal policy) or any other country characteristics that have been shown to correlate with macroeconomic volatility, such as per capita income, institutions, or industrial specialization. Column 2 includes GDP per capita. The fit of the model improves slightly, and though the coefficient on the model volatility is somewhat smaller, it remains significant at the 1 percent level. The next two columns include measures of export structure volatility and sectoral specialization, since di Giovanni and Levchenko (2009, 2012) show that opening to trade can affect aggregate volatility through changes in these variables. Column 3 adds the risk content of exports, which captures the overall riskiness of a country's export structure.[19] The

[19] This measure is taken from di Giovanni and Levchenko (2012). A country's export structure can be volatile because of a lack of diversification and/or exporting in sectors that are more volatile.

model volatility remains significant, and the R^2 of the regression is now .477. Finally, column 4 adds a measure of production specialization for the manufacturing sector (Herfindahl of sectoral production shares).[20] The number of observations drops to 35 because of limited data availability, but the model volatility still remains significant.

V. Robustness Checks and Model Perturbations

A. *Free Entry and Intermediate Inputs*

The assumption that the number of potential projects is determined by a free-entry condition may not be realistic. We thus simulate the quantitative model under the assumption that the numbers of potential entrepreneurs $\bar{I}_i^s$ are fixed in every country and sector.[21] Table 7 reports the results of this robustness check. For ease of comparison, the top row presents the two main results from the baseline analysis. The first is that the model generates higher volatility in smaller countries, with the elasticity of volatility with respect to country size of -0.135. (As reported above, in the data this elasticity is very close, -0.139.) The second key result of the paper is the contribution of trade openness to aggregate volatility. Column 2 reports the mean ratio of aggregate volatility under the current level of trade openness relative to complete autarky.

Row 2 of table 7 reports these two main results of the paper under the alternative assumption that $\bar{I}_i^s$ is fixed. Not surprisingly, the elasticity of volatility with respect to country size is virtually identical. Less obviously, the fixed-$\bar{I}_i^s$ model delivers very similar changes in volatility due to trade openness: the mean impact is 9.0 percent, compared to 9.7 percent with free entry.

A somewhat related question involves the role of intermediate input linkages. With intermediate inputs, trade opening reduces the costs of the input bundle faced by firms, making it easier to enter markets, all else equal. To assess the importance of this effect, we implement the baseline model with free entry but without intermediate input linkages: $\beta_T = \beta_N = 1$. The two main results are presented in the third row of table 7. The elasticity of volatility with respect to country size is only slightly larger than in the baseline, at -0.145. The impact of trade on volatility is much larger, at 23.8 percent.

[20] This measure is calculated using the UNIDO database of sectoral production and is taken from di Giovanni and Levchenko (2009).

[21] We set the values of $\bar{I}_i^s$ to be the same as in the free-entry baseline and adjust f_{ii} to match the 7 million operating firms in the United States in the trade equilibrium; the results are virtually the same if we instead adopt the common ad hoc assumption that $\bar{I}_i^s$ are some constant fraction of L_i, as in Chaney (2008), for instance.

TABLE 7
SENSITIVITY CHECKS: THE IMPACT OF COUNTRY SIZE AND TRADE ON VOLATILITY

	β_{Size} (1)	Trade/ Autarky (2)
Baseline	−.135	1.097
Fixed $\bar{I}_i^s$	−.133	1.090
$\beta_T = \beta_N = 1$	−.145	1.238
$\sigma = Ax^{-\xi}$	−.286	1.291
$\zeta = 1.5$	−.123	1.116
$\varepsilon = 4$	−.119	1.099
$\varepsilon = 8$	−.138	1.111

NOTE.—This table reports (1) the coefficient of regressing the log of aggregate volatility on the log of country size (β_{Size}) in the trade equilibrium and (2) the contribution of international trade to aggregate volatility (the mean ratio of volatility under trade to the volatility in autarky) under alternative assumptions. Row 1 reports the results for the baseline trade equilibrium simulation. Row 2 reports the results of a simulation without free entry. Row 3 reports the results of a simulation without intermediates. Row 4 reports the results of a simulation in which the firm-specific volatility decreases in firm size. Row 5 reports the results of applying a power law coefficient of 1.5 rather than the baseline of 1.06. Rows 6 and 7 report the results when using an elasticity of substitution of 4 or 8, respectively.

B. *Volatility Varying with Firm Size*

An assumption that simplifies the analysis above is that the volatility of the proportional change in sales, σ, does not change in firm size x. If the volatility of sales decreases sufficiently fast in firm size, larger firms will be so much less volatile that they will not affect aggregate volatility. In fact, an economy in which larger firms are just agglomerations of smaller units each subject to i.i.d. shocks is not granular: shocks to firms cannot generate aggregate fluctuations.

In practice, however, the negative relationship between firm size and its sales volatility is not very strong. Several papers estimate the relationship between size and volatility of the type $\sigma = Ax^{-\xi}$ using Compustat data (see, e.g., Stanley et al. 1996; Sutton 2002). The benchmark case in which larger firms are simply collections of independent smaller firms would imply a value of $\xi = 1/2$ and the absence of granular fluctuations. Instead, the typical estimate of this parameter is about $1/6$, implying that larger firms are not substantially less volatile than smaller ones.[22] Gabaix

[22] A related point concerns multiproduct firms: if large firms sell multiple imperfectly correlated products, then the volatility of the total sales for multiproduct firms will be lower than the volatility of single-product firms. Evidence suggests, however, that even in multiproduct firms the bulk of sales and exports is accounted for by a single product line. Sutton (2002) provides evidence that in large corporations, the constituent business units themselves follow a power law, with just a few very large business units and many much smaller ones. Along similar lines, Adalet (2009) shows that in the census of New Zealand firms, only

(2011) argues that these estimates may not be reliable since they are obtained using only data on the largest listed firms. In addition, it is not clear whether estimates based on the United States accurately reflect the experience of other countries. Hence, our baseline analysis sets $\xi = 0$ and a value of σ based on the largest 100 listed firms in the United States. In other words, we assume that all firms in the economy experience volatility as low as the largest firms in the economy.

To check robustness of our results, we allow the firm-specific volatility to decrease in firm size at the rate estimated in the literature. In that case, aggregate volatility is given by

$$\mathrm{SD}_z\left[\frac{\Delta X}{\mathrm{E}_z(X)}\right] = \sqrt{\sum_{k=1}^{I}\left[Ax(k)^{-\xi}h(k)\right]^2},$$

where, once again, $x(k)$ is sales of firm k, and $h(k)$ is the share of firm k's sales in total output in the economy.

The rest of the simulation remains unchanged. Since we are not matching the level of aggregate volatility, just the role of country size and trade, we do not need to posit a value of the constant A. However, it would be easy to calibrate to match the volatility of the top 100 firms in the United States as reported by Gabaix (2011), for example. Note that compared to the baseline simulation, modeling a decreasing relationship between firm size and volatility is a double-edged sword: while larger firms may be less volatile as a result, smaller firms are actually more volatile. This implies that the impact of either country size or international trade will not necessarily be more muted when we make this modification to the basic model.

Row 4 of table 7 reports the two main results of the paper under the alternative assumption that firm volatility decreases with firm size. In turns out that in this case, smaller countries are even more volatile relative to large ones (the size-volatility elasticity doubles to -0.286), and the contribution of trade is also larger, with trade leading to an average 29 percent increase in volatility compared to 9.7 percent in the baseline. Somewhat surprisingly, therefore, allowing volatility to decrease in firm size implies a larger contribution of trade to aggregate volatility, not a smaller one. In fact, this is the case in every country in the sample except the United States.[23]

about 6.5–9.5 percent of sales variation is explained by the extensive margin (more products per firm), with the rest explained by the intensive margin (greater sales per product).

[23] Another possible determinant of firm volatility that would be relevant to our analysis is exporting. The baseline model assumes that the volatility of a firm's sales growth does not change when it becomes an exporter. If exporters became systematically more or less volatile than nonexporters, the quantitative results could be affected. To check for this possibility, we used the Compustat quarterly database of listed US firms together with information on whether a firm is an exporter from the Compustat segments database. Appendix table D2 estimates the relationship between firm-level volatility—based on either

C. *Alternative Parameter Values*

We assess the sensitivity of the results in two additional ways. The first is an alternative assumption on the curvature of the firm size distribution. Eaton et al. (2011) estimate a range of values for $\theta/(\varepsilon-1)$ between 1.5 and 2.5. Though Gabaix (2011) shows that the shocks to large firms can still generate aggregate volatility when the power law exponent is less than two, it is important to check whether the main results of our paper survive under alternative values of $\theta/(\varepsilon-1)$. Row 5 of table 7 presents the two main results of the paper under the assumption that the slope of the power law in firm size is 1.5 instead of 1.06. Though in each case the numbers are slightly smaller in absolute value, the main qualitative and quantitative results remain unchanged: smaller countries still have lower volatility, with an elasticity of −0.123, and trade contributes slightly more to aggregate volatility, with an average increase of 11.6 percent.

Second, we recalibrate the model under two alternative values of ε, 4 and 8. In these exercises, we continue to assume that the economy is characterized by Zipf's law, so that $\theta/(\varepsilon-1)$ is still equal to our baseline value of 1.06. Thus, as we change ε, we change θ along with it. The results are presented in the last two rows of table 7. The size-volatility relationship is robust to these alternative assumptions. The elasticity of volatility with respect to country size is similar to the baseline, though slightly lower when $\varepsilon = 4$. The contribution of trade is quite similar as well, with 9.9 percent and 11.1 percent for $\varepsilon = 4$ and $\varepsilon = 8$, respectively.

Although for all of the robustness checks table 7 reports only the average impact of trade, it turns out that all of these alternative implementations preserve the basic patterns found in the baseline: trade raises volatility relative to autarky in all countries; larger countries and countries farther away from major trading partners tend to experience smaller changes in volatility due to trade.

D. *Further Reductions in Trade Costs*

The analysis above compares aggregate volatility under today's trade costs and in autarky and finds that the impact of trade on volatility has been robustly positive. We now evaluate how volatility would change if trade costs decreased further from their current levels. Table 8 presents the distribution of changes in aggregate volatility relative to its current

the growth rate of sales or a measure of the "granular residual" following Gabaix (2011)—and its export status and size. When we control for size, export status is always insignificant, and even the magnitude of the coefficient is exceedingly small, implying that volatility of exporters is between 96 and 99 percent of the volatility of nonexporters. Furthermore, the estimated elasticity of volatility with respect to firm size is similar to what is reported in the literature and used in the sensitivity check.

TABLE 8
THE IMPACT OF FURTHER REDUCTIONS IN TRADE COSTS ON AGGREGATE VOLATILITY

	REDUCTION IN TRADE COSTS			
	10% (1)	25% (2)	50% (3)	75% (4)
Percentile:				
5th	.998	.994	.984	1.003
10th	.998	.998	.991	1.006
25th	1.001	1.003	1.001	1.017
50th	1.004	1.011	1.011	1.034
75th	1.011	1.022	1.036	1.075
95th	1.019	1.036	1.055	1.129
Minimum	.995	.988	.973	.990
Maximum	1.030	1.050	1.084	1.167

NOTE.—This table reports percentiles and the minimum and maximum of the ratio of aggregate volatility under four reductions in iceberg trade costs τ_{ij} to the aggregate volatility as implied by the model under current trade costs.

level for various magnitudes of trade cost reductions, from 10 percent to 75 percent. Strikingly, a further reduction in trade costs leads to practically no change in volatility on average. For the median country, a 50 percent reduction in trade costs increases volatility by only 1.1 percent relative to the baseline. Furthermore, while the median volatility does rise slightly as trade costs fall, the impact always ranges from positive to negative.

What can explain this nonmonotonicity? Starting from autarky, as trade costs fall, only the largest firms export, and the distribution of firm size becomes more right-skewed. This is the main mechanism responsible for the positive effect of trade openness on volatility. However, as trade costs fall further, the exporting cutoff falls, and more and more firms begin exporting. Eventually, this process will make the firm size distribution less fat tailed: when trade costs are so low that everyone exports, there is no selection into exporting, and the power law in firm size exponent is the same as it was in autarky. Consistent with this intuition, we find that the change in aggregate volatility when trade costs fall is closely correlated with the concomitant change in the share of exporters. Smaller countries tend to experience the largest increases in the share of exporters and the greatest decreases in volatility in this counterfactual. The opposite is true for the biggest countries.

Two additional points are worth making about the impact of further reductions in trade costs. First, the nonmonotonicity is not due to the assumption of free entry: the results are virtually the same if we assume fixed $\bar{I}_i^s$ instead. Second, in the model without intermediates the nonmonotonicity disappears: reductions in trade costs always increase volatility in that model. The source of the difference is that without intermediates, a given fall in trade costs leads to a far smaller change in the exporting cutoffs than in the model with intermediates. Many fewer firms enter the export markets, and thus the selection into exporting effect is not reversed as it is in the baseline. What is the intuition for this difference? In both models, a global fall in τ_{ij} has the direct effect of lowering the firms' marginal cost of serving the export markets and thus makes the firm more likely to start selling abroad. In a model with intermediates, there is an additional effect that the fall in τ_{ij} lowers the cost of the input bundle c_j^s, since that input bundle includes foreign varieties and those are now cheaper. This indirect effect further lowers the exporting cutoffs, over and above the direct effect of τ_{ij}. In the model without intermediates the indirect effect is absent since the input bundle is just the wage. It turns out that quantitatively this makes a large difference for the results of a reduction in trade costs.

We conclude from these exercises that while the impact of openness on volatility at the current levels of trade costs is robustly positive in all the models we consider, the outcomes of further reductions in trade costs are sensitive to modeling assumptions about intermediate input linkages.

VI. Conclusion

Recent literature in both macroeconomics and international trade has focused attention on the role of large firms. Gabaix (2011) demonstrates that if the distribution of firm size follows a power law with an exponent close to negative one—which appears to be the case in the data—the economy is granular: shocks to the largest firms can lead to aggregate fluctuations.

This paper argues that the preponderance of large firms and their role in aggregate volatility can help explain two empirical regularities: (i) smaller countries are more volatile, and (ii) more open countries are more volatile. We calibrate and simulate a multicountry model of firm-level production and trade that can generate granular fluctuations. The model matches quite well a number of features of the data, such as observed bilateral and overall trade volumes, export participation ratios, and the relative size of the largest firms in different countries. We show that the model reproduces the elasticity of GDP volatility with respect to country size found in the data. The contribution of international trade to aggregate volatility varies a great deal depending on country characteristics. While it is minimal in large, relatively closed economies such as the United States or Japan, trade increases volatility by up to 15–20 percent in small open economies such as Denmark or Romania.

Recent research incorporates heterogeneous firms into fully dynamic general equilibrium macroeconomic models, focusing on the impact of persistent aggregate shocks and firm entry and exit (Ghironi and Melitz

2005; Alessandria and Choi 2007; Ruhl 2008). The importance of firm-specific idiosyncratic shocks for macroeconomic volatility via the granular channel emphasized in this paper should be viewed as complementary to this work. Future research incorporating these different mechanisms, as well as bringing disaggregated data to the models, will help provide an even more complete picture of the macroeconomic impact of trade integration.

Appendix A

Data Description and Sources

Data on total GDP, per capita income, and trade openness come from the World Bank's World Development Indicators database. Aggregate volatility is the standard deviation of the yearly growth rates of per capita GDP in constant local currency units over the period 1970–2006. Country size is the average share of the country's nominal US dollar GDP in the world US dollar GDP. Per capita income is the average real PPP-adjusted per capita GDP. All the averages are taken over the same period over which the volatility is computed, 1970–2006.

The figures on Fonterra are obtained from http://www.maf.govt.nz/mafnet/rural-nz/profitability-and-economics/contribution-of-land-based-industries-nz-economic-growth/contribution07.htm and http://tvnz.co.nz/view/page/423466/146647. The data on the 10 largest Korean business groups come from the Korean Development Institute courtesy of Wonhyuk Lim and are for the year 2006.

The data on the Herfindahl indices of firm sales and the size of the 10 largest and the single-largest firm come from ORBIS, a large multicountry database published by Bureau van Dijk that contains information on more than 50 million companies worldwide. The data come from a variety of sources, including, but not limited to, registered filings and annual reports. Importantly, the database includes both publicly traded and privately held firms. The main variable used in the analysis is total sales. For each country, we use the year with the most observations available, which is always between 2006 and 2008. ORBIS is the largest available nonproprietary firm-level database. Nonetheless, coverage is quite uneven across countries and years, implying that measures of concentration may not be reliable or comparable across countries. We alleviate this concern by restricting the sample of countries to those with a certain minimum number of firms and by using indices that are less prone to coverage-related biases. Di Giovanni and Levchenko (2013) present a more complete description of the ORBIS database and further evidence based on these data that firm size distributions in a large number of countries are extremely fat tailed.

To obtain values of τ_{ij}, we use the gravity estimates from the empirical model of Helpman et al. (2008). Combining geographical characteristics such as bilateral distance, common border, common language, whether the two countries are in a currency union, and others with the coefficient estimates reported by Helpman et al. yields, up to a multiplicative constant, the values of τ_{ij} for each country pair. We vary the multiplicative constant so as to match the mean and median imports/GDP ratios observed in the data in our sample of countries. Data on bilateral distance, common border, whether the country is an island or is landlocked, common language, and colonial ties are from Centre d'Etudes Prospectives et Informations Internationales. Data on legal origins come from La Porta et al. (1998). Finally, information on currency unions and free-trade areas come from Rose (2004), supplemented by Internet searches whenever needed. The advantage of the Helpman et al. estimates is that they are obtained in an empirical model that accounts explicitly for both fixed and variable costs of exporting and thus correspond most closely to the theoretical structure in our paper. Note that in this formulation, $\tau_{ij} = \tau_{ji}$ for all i and j.[24]

The values of f_{ii}^s and f_{ij}^s are calibrated following di Giovanni and Levchenko (2013). The World Bank's Doing Business Indicators database collects information on the administrative costs of setting up a firm—the time it takes, the number of procedures, and the monetary cost—in a large sample of countries in the world. The particular variable we use is the amount of time required to set up a business. We favor this indicator compared to others that measure entry costs either in dollars or in units of per capita income because in our model f_{ii}^s is a quantity of inputs rather than a value. We must normalize f_{ii}^s for one country. Thus, we proceed by setting $f_{\mathrm{US,US}}^s$ to a level just high enough to ensure an interior solution for production cutoffs.[25] Then, for every other country f_{ii}^s is set relative to the United States. To be precise, if, according to the Doing Business Indicators database, it takes 10 times longer to register a business in country i than in the United States, then $f_{ii}^s = 10 \times f_{\mathrm{US,US}}^s$. Since we do not have data on fixed costs of operating a business that vary by sector, we set f_{ii}^s to be equal in the N and T sectors.

To measure the fixed costs of international trade, we use the Trading across Borders module of the Doing Business Indicators. This module provides the costs of exporting a 20-foot dry-cargo container out of each country, as well as the costs of importing the same kind of container into each country. Parallel to our approach to setting the domestic cost f_{ii}^s, the indicators we choose are the amount of time required to carry out these transactions. This ensures that f_{ii}^T and f_{ij}^T are measured in the same units. We take the bilateral fixed cost f_{ij}^T to be the sum of the cost of exporting from country j and the cost of importing into country i. The foreign trade costs f_{ij}^T are on average about 40 percent of the domestic entry costs f_{ii}^T.[26]

[24] An earlier version of the paper also computed τ_{ij} using the estimates of Eaton and Kortum (2002) as a robustness check. The results were very similar.

[25] That is, we set $f_{\mathrm{US,US}}^s$ to a level just high enough that $a_{ji}^s < 1/b_s$ for all $i, j = 1, \ldots, \mathcal{C}$ in all the baseline and counterfactual exercises, with $1/b_s$ being the upper limit of the distribution of a.

[26] An earlier version of the paper was more agnostic about the nature of domestic fixed costs f_{ii}^T and assumed instead that they are equal (and low) in every country. The results were very similar. In addition, we carried out the analysis setting the bilateral fixed cost to be the sum of domestic costs of starting a business in the source and destination countries: $f_{ij}^T = f_{ii}^T + f_{jj}^T$. This approach may be preferred if fixed costs of exporting involved more than just shipping and required, for instance, the exporting firm to create a subsidiary for the distribution in the destination country. The results were virtually identical.

The bilateral and overall trade volumes as a share of GDP used for comparison to the model come from the International Monetary Fund's Direction of Trade Statistics.

Appendix B

The Complete Two-Sector Model

In country i, consumers maximize

$$\max_{\{y_i^N(k), y_i^T(k)\}} \left[\sum_{k=1}^{J_i^N} y_i^N(k)^{(\varepsilon_N-1)/\varepsilon_N}\right]^{\alpha\varepsilon_N/(\varepsilon_N-1)} \left[\sum_{k=1}^{J_i^T} y_i^T(k)^{(\varepsilon_T-1)/\varepsilon_T}\right]^{(1-\alpha)\varepsilon_T/(\varepsilon_T-1)}$$

subject to

$$\sum_{k=1}^{J_i^N} p_i^N(k) y_i^N(k) + \sum_{k=1}^{J_i^T} p_i^T(k) y_i^T(k) = Y_i,$$

where $y_i^s(k)$ is final consumption of good k belonging to sector $s = N, T$ in country i; $p_i^s(k)$ is the price of this good; Y_i is total final consumption expenditure in the economy; and J_i^s is the number of varieties available in sector s in country i coming from all countries. Since consumer preferences are Cobb-Douglas in CES aggregates of N and T, final consumption expenditure on sector N is equal to αY_i and on the T sector, $(1-\alpha)Y_i$.

The CES composites of both N and T are used both as consumption and as intermediate inputs in production. Let X_i^s denote the total spending—final and intermediate—on sector $s = N, T$ in country i. Given this total expenditure, it is well known that expenditure on an individual variety k in country i is equal to

$$x_i^s(k) = \frac{X_i^s}{(P_i^s)^{1-\varepsilon_s}} p_i^s(k)^{1-\varepsilon_s},$$

where P_i^s is the ideal price index of sector s in this economy, (2), augmented with the appropriate sector superscripts.

Production in both sectors uses both labor and CES composites of N and T as intermediate inputs. An input bundle in country j and sector s has a cost

$$c_j^s = w_j^{\beta_s}[(P_j^N)^{\eta_s}(P_j^T)^{1-\eta_s}]^{1-\beta_s}.$$

That is, production in sector $s = N, T$ requires labor, inputs of N, and inputs of T. The share of labor in value added, β_s, and the share of nontradable inputs in total input usage, η_s, both vary by sector.

Entrepreneurs can pay the exploration cost f_e to enter either sector. Each entrepreneur that entered sector s in country j decides whether or not to pay the fixed cost of production f_{ij}^s and which, if any, export markets to serve. In the N sector, we assume that trade costs are infinite, and thus a firm in country j may serve only its own market.

The expressions defining the input requirement cutoffs a_{ij}^s, (3) and (4), and the free-entry conditions (5) have the same form (up to the appropriate sector superscripts) and will hold in each sector. Following similar steps, we derive the expressions for the price levels in the two sectors:

$$P_i^N = \frac{1}{b_N}\left[\frac{\theta_N}{\theta_N-(\varepsilon_N-1)}\right]^{-1/\theta_N}\frac{\varepsilon_N}{\varepsilon_N-1}\left(\frac{X_i^N}{\varepsilon_N}\right)^{-[\theta_N-(\varepsilon_N-1)]/[\theta_N(\varepsilon_N-1)]} \times\left\{\bar{I}_i^N\left(\frac{1}{c_i^N}\right)^{\theta_N}\left(\frac{1}{c_i^N f_{ii}^N}\right)^{[\theta_N-(\varepsilon_N-1)]/(\varepsilon_N-1)}\right\}^{-1/\theta_N} \tag{B1}$$

and

$$P_i^T = \frac{1}{b_T}\left[\frac{\theta_T}{\theta_T-(\varepsilon_T-1)}\right]^{-1/\theta_T}\frac{\varepsilon_T}{\varepsilon_T-1}\left(\frac{X_i^T}{\varepsilon_T}\right)^{-[\theta_T-(\varepsilon_T-1)]/[\theta_T(\varepsilon_T-1)]} \times\left\{\sum_{j=1}^{\mathcal{C}}\bar{I}_j^T\left(\frac{1}{\tau_{ij}c_j^T}\right)^{\theta_T}\left(\frac{1}{c_j^T f_{ij}^T}\right)^{[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)}\right\}^{-1/\theta_T}. \tag{B2}$$

Having expressed P_i^s and a_{ij}^s in terms of X_i^s and c_i^s for all $i, j = 1, \ldots, \mathcal{C}$, it remains to close the model by solving for the X_i^s's and w_i's. To do this, we impose balanced trade for each country and the market-clearing conditions in each sector and country. Free entry implies that the total profits are zero, and thus final expenditure in country i simply equals labor income: $Y_i = w_i L_i$. Total expenditure X_i^N and X_i^T equal final spending plus expenditure on sector s as intermediate inputs in both sectors:

$$X_i^N = \alpha w_i L_i + (1-\beta_N)\eta_N X_i^N + (1-\beta_T)\eta_T X_i^T,$$

$$X_i^T = (1-\alpha)w_i L_i + (1-\beta_N)(1-\eta_N)X_i^N + (1-\beta_T)(1-\eta_T)X_i^T.$$

Note that even though the T sector has both imports and exports, the assumption that only T-sector goods can be traded amounts to imposing balanced trade within the T sector, and thus the second condition must be satisfied in equilibrium as written. These two conditions imply that total spending in each sector is a constant multiple of labor income $w_i L_i$.

Total sales from country i to country j can be written as

$$X_{ji}^T = \frac{X_j^T}{(P_j^T)^{1-\varepsilon_T}}\left(\frac{\varepsilon_T}{\varepsilon_T-1}\tau_{ji}c_i^T\right)^{1-\varepsilon_T}\bar{I}_i^T\frac{b_T^{\theta_T}\theta_T}{\theta_T-(\varepsilon_T-1)}(a_{ji}^T)^{\theta_T-(\varepsilon_T-1)}.$$

Using expressions for a_{ji}^T in (4) and P_j^T in (B2), total exports from i to j become

$$X_{ji}^T = \frac{\bar{I}_i^T(\tau_{ji}c_i^T)^{-\theta_T}(f_{ji}^T c_i^T)^{-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)}}{\sum_{l=1}^{\mathcal{C}}\bar{I}_l^T(\tau_{jl}c_l^T)^{-\theta_T}(f_{jl}^T c_l^T)^{-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)}} X_j^T.$$

Using the trade balance conditions, $X_i^T = \sum_{j=1}^{\mathcal{C}} X_{ji}^T$ for each $i = 1, \ldots, \mathcal{C}$ as well as the property that total spending X_i^T is a constant multiple of $w_i L_i$ leads to the following system of equations in w_i:

$$\begin{aligned} w_i L_i = \sum_{j=1}^{\mathcal{C}} \Bigg[\Bigg(& \bar{I}_i^T \tau_{ji}^{-\theta_T} (f_{ji}^T)^{-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)} \\ & \times \{ w_i^{\beta_T} [(P_i^N)^{\eta_T}(P_i^T)^{1-\eta_T}]^{1-\beta_T} \}^{-\theta_T-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)} \Bigg) \\ & \div \Bigg(\sum_{l=1}^{\mathcal{C}} \bar{I}_l^T \tau_{jl}^{-\theta_T} (f_{jl}^T)^{-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)} \\ & \times \{ w_l^{\beta_T} [(P_l^N)^{\eta_T}(P_l^T)^{1-\eta_T}]^{1-\beta_T} \}^{-\theta_T-[\theta_T-(\varepsilon_T-1)]/(\varepsilon_T-1)} \Bigg) \Bigg] w_j L_j, \end{aligned} \tag{B3}$$

$i = 1, \ldots, \mathcal{C}$. There are $\mathcal{C} - 1$ independent equations in this system, with wage in one of the countries as the numeraire.

A monopolistically competitive equilibrium is a set of prices $\{w_i, P_i^N, P_i^T\}_{i=1}^{\mathcal{C}}$ and factor allocations such that (i) consumers maximize utility, (ii) firms maximize profits, and (iii) all goods and factor markets clear. The equilibrium is obtained as a solution to $(\mathcal{C} - 1) + 2 \times \mathcal{C} + 2 \times \mathcal{C}$ equations in w_i, P_i^N, P_i^T, $\bar{I}_i^N$, and $\bar{I}_i^T$ that satisfies equations (5) for both $s = N, T$, (B1), (B2), and (B3) for each $i = 1, \ldots, \mathcal{C}$. We solve these equations numerically in order to carry out the main quantitative exercise in this paper.

Calibrating to Zipf's law in firm size in a two-sector model with trade.—While Section III.B argues that, in a one-sector Melitz-Pareto economy, steady-state firm size follows a power law with exponent $\theta/(\varepsilon - 1)$, our quantitative model features two sectors, idiosyncratic shocks to firm sales and selection into exporting. Here we show that the aggregate model economy with these additional features will still exhibit Zipf's law in firm size.

Deriving an aggregate power law in an economy with two sectors involves computing the (counter-)cdf of the following mixture of distributions. Let Q be a random variable that follows a power law with exponent ζ_1 with probability p and with exponent ζ_2 with probability $1 - p$. It is straightforward to show that the counter-cdf of Q is equal to $\Pr(Q > q) = pD_1 q^{-\zeta_1} + (1 - p)D_2 q^{-\zeta_2}$. Importantly, when $\zeta_1 = \zeta_2 = \zeta$, Q is itself a power law with exponent ζ. This means that a two-sector economy in which both sectors follow a power law with the same exponent will, in aggregate, also exhibit a power law with that exponent. Our quantitative exercise adopts the assumption that both the N and T sectors follow Zipf's law. Though we are not aware of any comprehensive set of estimates of power law exponents in both traded and nontraded sectors, di Giovanni et al. (2011) estimate power law exponents for a wide range of both traded and nontraded industries using a census of French firms and find that power law exponents do not differ systematically between traded and nontraded sectors. It still could be the case that while the reduced-form exponents—which correspond to $\theta_T/(\varepsilon_T - 1)$ and $\theta_N/(\varepsilon_N - 1)$—are the same, the actual values of θ_s and ε_s differ. Since we do not have reliable information about how these two individual parameters differ across sectors, we adopt the most agnostic and neutral assumption that both θ_s and ε_s are the same in the two sectors.

Another concern is that even if steady-state firm size in the aggregate economy follows Zipf's law, when firms are hit by idiosyncratic shocks z, the resulting distribution would be something else. It turns out, however, that power laws are preserved under multiplication by a random variable with finite variance. That is, if firm sales are driven by a random productivity that generates Zipf's law ($1/a$ in our notation) and a finite variance shock (z), the resulting distribution of sales is still Zipf (Gabaix 2009, 258–59).

Another point regarding the calibration of power law parameters is that, strictly speaking, when not all firms export, selection into exporting implies that the power law exponent estimated on total sales—domestic plus exporting—is lower than $\theta/(\varepsilon - 1)$. Di Giovanni et al. (2011) explore this bias in detail using the census of French firms and suggest several corrections to the estimating procedure that can be used to estimate $\theta/(\varepsilon - 1)$ in an internally consistent way. Their analysis shows that the bias introduced by selection into exporting is not large. Corrected estimates obtained by di Giovanni et al. show that $\theta/(\varepsilon - 1)$ is about 1.05, roughly the same as the value used in the quantitative exercise.

Thus, even though the model is enriched with these additional features, the resulting distribution of firm size that the model produces still follows Zipf's law.

Appendix C

Aggregate Volatility Derivation

Firm k in country i with unit input requirement $a(k)$ and realization of transitory shock $z(k)$ has sales of

$$\begin{aligned} x^s(a(k), z(k)) &= \sum_{j=1}^{\mathcal{C}} \mathbf{1}[a(k) \le a_{ij}] \frac{X_j^s}{(P_j^s)^{1-\varepsilon_s}} \left[\frac{\varepsilon_s}{\varepsilon_s - 1} \tau_{ji} c_i^s a(k) z(k)\right]^{1-\varepsilon_s} \\ &= \left\{\sum_{j=1}^{\mathcal{C}} \mathbf{1}[a(k) \le a_{ij}] \frac{X_j^s}{(P_j^s)^{1-\varepsilon_s}} \left[\frac{\varepsilon_s}{\varepsilon_s - 1} \tau_{ji} c_i^s a(k)\right]^{1-\varepsilon_s}\right\} \tilde{z}, \end{aligned} \tag{C1}$$

where $\mathbf{1}[\cdot]$ is the indicator function that captures whether firm k serves market j, and $\tilde{z} \equiv z^{1-\varepsilon_s}$. We already assumed that $\mathrm{E}_z(\tilde{z}) = 1$, and now we further suppose that $\mathrm{Var}_x(\tilde{z}) = \sigma^2$. Expected sales for the firm with productivity $a(k)$ are

$$\mathrm{E}_z[x^s(a(k), z(k))] = \sum_{j=1}^{\mathcal{C}} \mathbf{1}[a(k) \le a_{ij}] \frac{X_j^s}{(P_j^s)^{1-\varepsilon_s}} \left[\frac{\varepsilon_s}{\varepsilon_s - 1} \tau_{ji} c_i^s a(k)\right]^{1-\varepsilon_s}. \tag{C2}$$

Given the expression for the actual sales of the firm with a transitory shock $z(k)$ in (C1) and the expected sales of the firm with productivity $a(k)$ in (C2), the actual sales as an approximation around $\mathrm{E}_z[x^s(a(k), z(k))]$ are

$$x^s(a(k), z(k)) \approx \mathrm{E}_z[x^s(a(k), z(k))] + \left.\frac{dx}{d\tilde{z}}\right|_{\tilde{z}=1} \Delta\tilde{z}.$$

Therefore, the proportional change in $x^s(a(k), z(k))$, or the growth rate, is given by

$$\frac{\Delta x^s(a(k), z(k))}{\mathrm{E}_z[x^s(a(k), z(k))]} = \tilde{z} - 1,$$

and the variance of this growth rate is

$$\mathrm{Var}_z\left\{\frac{\Delta x^s(a(k), z(k))}{\mathrm{E}_z[x^s(a(k), z(k))]}\right\} = \sigma^2,$$

which we assume for simplicity is the same in the two sectors $s = N, T$. When we drop the sector superscripts, the total sales in the economy are given by (8); thus the change in the total sales relative to the nonstochastic steady state (the growth rate) is

$$\begin{aligned}\frac{\Delta X}{\mathrm{E}_z X} &= \frac{\sum_{k=1}^{I} \Delta x(a(k), z(k))}{\mathrm{E}_z X} \\ &= \sum_{k=1}^{I} \frac{\Delta x(a(k), z(k))}{\mathrm{E}_z[x(a(k), z(k))]} \frac{\mathrm{E}_z[x(a(k), z(k))]}{\mathrm{E}_z X}.\end{aligned}$$

This means that the aggregate volatility is

$$\begin{aligned}\mathrm{Var}_z\left(\frac{\Delta X}{\mathrm{E}_z X}\right) &= \mathrm{Var}_z\left\{\sum_{k=1}^{I} \frac{\Delta x(a(k), z(k))}{\mathrm{E}_z[x(a(k), z(k))]} \frac{\mathrm{E}_z[x(a(k), z(k))]}{\mathrm{E}_z X}\right\} \\ &= \sum_{k=1}^{I} \mathrm{Var}_z\left\{\frac{\Delta x(a(k), z(k))}{\mathrm{E}_z[x(a(k), z(k))]}\right\}\left\{\frac{\mathrm{E}_z[x(a(k), z(k))]}{\mathrm{E}_z X}\right\}^2 \\ &= \sigma^2 \sum_{k=1}^{I}\left\{\frac{\mathrm{E}_z[x(a(k), z(k))]}{\mathrm{E}_z X}\right\}^2 \\ &= \sigma^2 \sum_{k=1}^{I} h(k)^2,\end{aligned}$$

where $h(k)$ is the share of the firm k's expected sales in total expected sales in the economy. As expected, the volatility of total output in the economy is equal to the volatility of an individual firm's output times the Herfindahl index of production shares.

Appendix D

TABLE D1
AGGREGATE VOLATILITY AND COUNTRY SIZE REGRESSIONS

	Dependent Variable: Log(GDP Volatility)					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(size)	−.177**	−.139**	−.090+	−.209**	−.180**	−.142**
	(.038)	(.044)	(.045)	(.035)	(.027)	(.023)
Log(GDP per capita)		−.157*	−.261**	−.049	−.019	.018
		(.069)	(.070)	(.057)	(.045)	(.037)
Constant	−4.352**	−2.696**	−1.533+	−4.010**	−4.154**	−4.291**
	(.190)	(.763)	(.773)	(.601)	(.473)	(.410)
Observations	49	49	30	75	100	143
R^2	.192	.273	.337	.328	.296	.225

NOTE.—The dependent variable is the log of the standard deviation of per capita GDP growth over the period 1970–2006. Size is a country's GDP as a share of world GDP; GDP per capita is PPP-adjusted per capita income. All right-hand-side variables are averages over 1970–2006. Robust standard errors are in parentheses.
+ Significant at 10 percent.
* Significant at 5 percent.
** Significant at 1 percent.

TABLE D2
US EVIDENCE ON RELATIONSHIP BETWEEN FIRM-LEVEL VOLATILITY AND EXPORTER STATUS AND SIZE

	Growth		Granular	
	All (1)	Restricted (2)	All (3)	Restricted (4)
	A. Sample Period: 1980–2007			
Exporter	−.022	−.014	−.024	−.017
	(.021)	(.022)	(.021)	(.023)
Log(sales)	−.129**	−.135**	−.128**	−.133**
	(.005)	(.005)	(.005)	(.004)
Observations	15,901	14,597	15,859	14,558
Number of SIC	440	415	427	403
R^2	.181	.183	.198	.201
	B. Sample Period: 1980–89			
Exporter	−.020	−.013	−.010	−.013
	(.021)	(.023)	(.020)	(.023)
Log(sales)	−.128**	−.133**	−.126**	−.133**
	(.005)	(.006)	(.004)	(.006)
Observations	8,529	7,693	8,509	7,693
Number of SIC	435	410	422	410
R^2	.171	.170	.181	.170
	C. Sample Period: 1990–2007			
Exporter	−.025	−.021	−.041	−.036
	(.029)	(.031)	(.029)	(.031)
Log(sales)	−.136**	−.140**	−.134**	−.140**
	(.008)	(.008)	(.008)	(.007)

TABLE D2 (*Continued*)

	Growth		Granular	
	All (1)	Restricted (2)	All (3)	Restricted (4)
Observations	6,881	6,467	6,857	6,443
Number of SIC	409	386	398	375
R^2	.149	.151	.165	.174

Note.—This table presents the results from regressing a measure of firm-level sales volatility on measures of its export status (exporter) and size (sales). Columns 1 and 2 (growth) take the natural logarithm of the standard deviation of firm real sales as the dependent variable. Columns 3 and 4 (granular) use a granular volatility measure, calculated as the standard deviation of the estimated residuals, $\hat{\varepsilon}_{ist}$, from the following firm-level panel regression: $\Delta\log(\text{sales}_{ist}) = \alpha_i + \alpha_{st} + \varepsilon_{ist}$, where i is a firm, s is a sector, and t is a quarter, so that α_i is a firm-level effect and α_{st} is a sector × time effect. Standard deviations are calculated over the given sample period, while export status and measures of firm size are averaged over the period. Regressions include sector-level fixed effects at the four-digit Standard Industrial Classification. All includes all firms, while restricted excludes firms in the commodity, energy, and public sectors. Data are taken from the Compustat Quarterly database of listed US firms together with information on whether a firm is an exporter from the Compustat Segments database. Robust standard errors are in parentheses.

$^+$ Significant at 10 percent.

* Significant at 5 percent.

** significant at 1 percent.

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