Country Size, International Trade, and Aggregate Fluctuations in Granular Economies
WEB APPENDIX*

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Web Appendix A Impact of Country Size and Openness in a Canonical IRBC Model

This Appendix evaluates whether the standard International Real Business Cycle (IRBC) model produces the relationship between country size and volatility observed in the data, and whether it features a positive relationship between openness and volatility. To answer these questions, we implement the standard IRBC model and vary relative country size and openness, focusing on the response of aggregate volatility to those two variables. The modeling approach and calibration details follow as closely as possible the classic treatment of Backus et al. (1995, henceforth BKK).

A.1 Country Size

Let there be two economies, Home and Foreign, with Foreign variables denoted by an asterisk. The time horizon is infinite and indexed by t. In the Home country, each agent's utility depends on consumption and leisure:

$$U = \sum_{t=0}^{\infty} \gamma^t \frac{[c_t^{\mu}(1-n_t)^{1-\mu}]^{1-\eta} - 1}{1-\eta},$$

where c_t is an individual's consumption, and n_t is the share of the time endowment dedicated to working in period t. In order to model countries of differing sizes, we follow the approach

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of Head (1995) and Crucini (1997) and assume that Home is populated by $\mathcal{T}$ identical agents, and Foreign is populated by $\mathcal{T}^*$. Adding up utilities of all agents in Home, we obtain how total welfare in period t depends on the aggregate consumption C_t and total labor supply N_t:

$$U(C_t, N_t) = \mathcal{T}u(c_t, n_t) = \mathcal{T}\left(\frac{[c_t^{\mu}(1-n_t)^{1-\mu}]^{1-\eta}-1}{1-\eta}\right)$$
$$= \mathcal{T}\left\{\frac{\left[\left(\frac{C_t}{\mathcal{T}}\right)^{\mu}\left(\frac{\mathcal{T}-N_t}{\mathcal{T}}\right)^{1-\mu}\right]^{1-\eta}-1}{1-\eta}\right\}$$
$$= \frac{\mathcal{T}^{\eta}[C_t^{\mu}(\mathcal{T}-N_t)^{1-\mu}]^{1-\eta}-\mathcal{T}}{1-\eta},$$

with the analogous aggregation in Foreign.

Production uses both labor and capital. Total output in Home is given by the Cobb-Douglas production function: $Y_t = Z_t K_{t-1}^{\rho} N_t^{1-\rho}$, where Z_t is aggregate productivity, and K_{t-1} is the capital stock available for production at the beginning of period t. Capital accumulation is subject to standard quadratic adjustment costs: investment of I_t in period t has the adjustment cost equal to $\frac{\phi}{2}\frac{(I_t-\delta K_{t-1})^2}{K_{t-1}}$, where δ is the depreciation rate. That is, it is costless to invest to cover depreciation exactly, but deviations from that incur an additional cost.

Finally, (log) productivity follows a bivariate AR(1) process:

$$\begin{bmatrix} \log Z_t \\ \log Z_t^* \end{bmatrix} = \begin{bmatrix} (1-\psi)\log \bar{Z} \\ (1-\psi)\log \bar{Z}^* \end{bmatrix} + \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \times \begin{bmatrix} \log Z_{t-1} \\ \log Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}.$$

Following BKK, Head (1995), and Crucini (1997), we assume that there is a single good, produced in both countries, that is used for both consumption and investment. Markets are complete, and thus equilibrium is found by solving a global planning problem that maximizes the net present value of world welfare subject to the global resource constraint:

$$C_t + C_t^* + K_t - (1-\delta)K_{t-1} + \frac{\phi}{2}\frac{(K_t - K_{t-1})^2}{K_{t-1}} + K_t^* - (1-\delta)K_{t-1}^* + \frac{\phi}{2}\frac{(K_t^* - K_{t-1}^*)^2}{K_{t-1}^*}$$
$$\leq Z_t K_{t-1}^{\rho} N_t^{1-\rho} + Z_t^* K_{t-1}^{*\rho} N_t^{*1-\rho},$$

that is, global output $Z_t K_{t-1}^{\rho} N_t^{1-\rho} + Z_t^* K_{t-1}^{*\rho} N_t^{*1-\rho}$ is used for consumption in the two countries, plus investment inclusive of adjustment costs.

With the exception of the differing country sizes, the model is standard, and is solved using conventional techniques of a first-order approximation around a deterministic steady state. In choosing parameter values, we follow BKK: the discount rate is set to $\gamma = 0.99$;

risk aversion/intertemporal elasticity of substitution $\eta = 2$; the weight of consumption in utility $\mu = 0.5$, depreciation rate $\delta = 0.025$, capital share $\rho = 0.36$, the persistence of the technology shock $\psi_{11} = \psi_{22} = 0.906$, the spillover terms $\psi_{12} = \psi_{21} = 0.088$ and the adjustment cost of investment parameter $\phi = 8.5$. With identical country sizes, the model is exactly the same as in BKK, and we do not discuss its properties here.

We simulate the model for a range of relative country sizes intended to mimic the size of countries relative to the rest of the world found in our data: from two countries accounting for 0.5 of the world GDP at one extreme to one country accounting for 2.5% of world GDP and the other for 97.5% of world GDP (in our data, the smallest countries are about 2% of world GDP). For each pair of relative country sizes, we draw random shocks ϵ_t and ϵ_t^* for 80,100 periods, and calculate the volatility of GDP growth for the last 80,000 periods (discarding the first 100 "burn-in" periods). Using these model-generated volatilities, we run the same regression as we do with the actual data, regressing log standard deviation of GDP growth on log share of the country in the world GDP. Consistent with previous findings (Head, 1995; Crucini, 1997), we do find that smaller countries are more volatile, but the elasticity of volatility with respect to country size is -0.004, two orders of magnitude lower than the volatility found in the data (-0.139), or the granular model in this paper (-0.135). We conclude that the standard IRBC model is not as successful as our model at generating the size-volatility relationship observed in the data.

A.2 Trade Openness

Because the one-good model above has steady state imports/GDP equal to zero, in order to evaluate the relationship between observed openness (trade/GDP) and volatility in the standard IRBC model, we augment it to feature Armington aggregation between domestic and foreign goods. That is, Home consumption and investment inclusive of adjustment costs comes from CES aggregation of domestic and foreign intermediate inputs:

$$C_t + K_t - (1-\delta)K_{t-1} + \frac{\phi}{2}\frac{(K_t - K_{t-1})^2}{K_{t-1}} = \left[\omega^{\frac{1}{\nu}}\left(y_t^h\right)^{\frac{\nu-1}{\nu}} + (1-\omega)^{\frac{1}{\nu}}\left(y_t^f\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \text{(A.1)}$$

where y_t^h is the output of the Home intermediate good that is used in Home production, and y_t^f is the amount of the Foreign intermediate used in Home production. In this standard formulation, consumption and investment are perfect substitutes, and Home and Foreign goods are aggregated in a CES production function. Domestic output is then divided between domestic intermediate inputs y_t^d and exports y_t^h:

$$Y_t = y_t^d + y_t^f = Z_t K_{t-1}^{\rho} N_t^{1-\rho}.$$

The rest of the model and calibration details remain unchanged. The key parameter is the Armintgon elasticity of substitution between domestic and foreign goods ν. We perform our simulation for two standard values of this elasticity, the "classic" BKK value of 1.5, and the alternative value suggested by BKK, and used in the subsequent literature of 0.9, implying that domestic and foreign goods are complements. Once again, we do not discuss the details of the model's performance, as it is highly standard. Instead, we simulate the model for a range of values of trade openness. Following BKK, we use the parameter ω – home bias in preferences – to vary trade openness. As it happens, the steady state ratio of imports to GDP corresponds exactly to ω. Thus, we solve the model for a range of values of ω between 0.01 to 0.5, the latter being the case of no home bias. Similarly to the previous exercise, we then simulate the model for 80,100 periods, recording the volatility of the growth rate of output for each level of trade openness. It turns out that the sign of the openness-volatility relationship depends crucially on the Armington elasticity ν. For $\nu = 1.5$, output volatility actually decreases in openness, with the elasticity of standard deviation of output growth with respect to imports/GDP of -0.029. However, under $\nu = 0.9$, volatility increases in openness, with an elasticity of similar order of magnitude but reverse sign, 0.085. Thus, it appears that there is no "natural" openness-volatility relationship in the standard IRBC framework; instead, the direction of the relationship depends a great deal on key parameter values.

Web Appendix B Endogenous Markups

The model in the main text is solved under the assumption that each firm treats the sectoral price level as given setting its prices and maximizing profits. This assumption is adopted in the overwhelming majority of the trade under monopolistic competition literature that followed Krugman (1980) and Melitz (2003), and leads to the well-known property of constant markup over marginal cost. However, because our paper emphasizes the role of extremely large firms in the economy, it is important to assess whether, and to what extent, price setting by the large firms departs from the constant-markup benchmark. Unfortunately, fully taking this phenomenon into account would be impractical. To incorporate this feature into the solution of the model would lead us to lose all of the analytical results that help solve the model, such as the expressions for the price levels and sales. This is because each firm's profit-maximizing price is a function of all the other firms' prices, so that just to pin down a single firm's price, quantity, total sales, and profits, we would have to solve a fixed point problem involving all the firms selling to that market. In the trade equilibrium, to do this while at the same time solving for wages and imposing a free-entry condition that pins down the equilibrium number of firms would not be feasible.

However, in some simpler cases we can check whether this phenomenon is quantitatively important. In this Appendix, we solve for the individual firms' prices and the aggregate price level in the autarky equilibrium for any particular draw of productivities. We start by finding the profit-maximizing price for each firm taking the prices of all the other firms as given. We then take all the firms' prices, and use those as the next starting point for finding each firm's profit maximizing price. Iterating to convergence, we obtain the full set of equilibrium prices for each firm, as well as the overall price levels in this economy. We then compare those to the individual firm prices and sectoral price levels that we would get if we instead assumed the constant markup equal to $\varepsilon/(\varepsilon-1)$.

Solving the firm's profit maximization problem, firm k's optimal price $p(k)$ is given implicitly by the following expression:

$$p(k) = \underbrace{\frac{\varepsilon}{\varepsilon-1}ca(k)}_{\text{“constant-markup price”}} \times \underbrace{\frac{1}{1-\frac{p(k)^{1-\varepsilon}}{\sum_{l=1}^{J}p(l)^{1-\varepsilon}}\left(1-\frac{ca(k)}{p(k)}\right)}}_{\text{general equilibrium contribution}}, \tag{B.1}$$

where c is the cost of the input bundle, $a(k)$ is the firm's unit input requirement and there are J firms in this market in total. The first term is the simple “constant-markup” price that is used in the large majority of the literature. The second term is the adjustment due to the firm's impact on the overall price level.

This equation does not have an analytical solution in $p(k)$, making it necessary to resort to numerical simulations. We thus take each country's autarky equilibrium number of firms, draw productivity for each firm, and solve for a fixed point in prices when each firm sets its price according to equation (B.1). Note that while this does not represent the full solution to the autarky model, it does allow us to evaluate how much flexible-markup prices differ from the constant-markup prices *for the constant-markup equilibrium number of firms.*

It turns out that in our sample of 50 countries, the maximum proportional difference between the flexible-markup price and the simplistic constant-markup price is 0.0225 (not percentage points of markup!). This is the maximum over all the firms in all the countries and all the sectors $s = N, T$. Thus, it appears that even for the very largest firms in the smallest countries, the constant-markup case is a very good approximation of their pricing behavior.

When it comes to the aggregate price levels, this phenomenon is even less important. Among the 50 countries and the 2 sectors (N and T) in our model, the maximum proportional difference in the price level is 0.4% (0.004). Thus, the general equilibrium effect whereby a large firm will take into account the impact of its own price on the aggregate price level appears to be quite minor quantitatively.

What is the intuition for this result? Though the flexible-markup price (B.1) does not have an analytical solution, it can be approximated as follows. The term $p(k)^{1-\varepsilon}/\sum_{l=1}^{J} p(l)^{1-\varepsilon}$ roughly corresponds to the share of the firm in total sales in the market. The term $ca(k)/p(k)$ is the inverse of the markup over the marginal cost, which we will approximate by $(\varepsilon-1)/\varepsilon$. The price then becomes, approximately:

$$p(k) \approx \underbrace{\frac{\varepsilon}{\varepsilon-1}ca(k)}_{\text{``constant-markup price''}} \times \underbrace{\frac{\varepsilon}{\varepsilon - share(k)}}_{\text{general equilibrium contribution}},$$

where $share(k)$ is the share of firm k in total sales in the market. That second term is thus the proportional deviation of the actual profit-maximizing price from the simplistic constant-markup price. How large does the firm have to be before this adjustment starts to appreciably affect the price? Under the baseline value of $\varepsilon = 6$, when $share(k)$ is 5%, the $\varepsilon/(\varepsilon - share(k))$ adjustment amounts to 1.0084; when $share(k)$ is 10%, that adjustment is 1.0169, and when $share(k)$ is 50%, the adjustment is 1.0909. Thus, even for firms that capture 50% of the total sales in a market, we are no more than 10% off if we simply assume the constant-markup price. As described above, given the number of firms that we actually draw for each country, we are never more than 0.0225 off for any given firm in the world.

This derivation can be used to make an additional point. While we perform this simulation for the autarky case, under trade these price deviations will become strictly smaller. This is because after trade opening, the market share of each firm *in any given market* will be strictly lower than in autarky, due to the appearance of foreign varieties (this result is well known in the heterogeneous firms literature). This in turn implies that the markup adjustment term $\varepsilon/(\varepsilon - share(k))$ will shrink further under trade.

References

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, "International Business Cycles: Theory and Evidence," in Thomas Cooley, ed., *Frontiers of Business Cycle Research*, Princeton: Princeton University Press, 1995, pp. 331–356.

Crucini, Mario J., "Country Size and Economic Fluctuations," *Review of International Economics*, May 1997, *5* (2), 204–220.

Head, Allen C., "Country Size, Aggregate Fluctuations, and International Risk Sharing," *Canadian Journal of Economics*, November 1995, *28* (4b), 1096–1119.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 1980, *70* (5), 950–959.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.